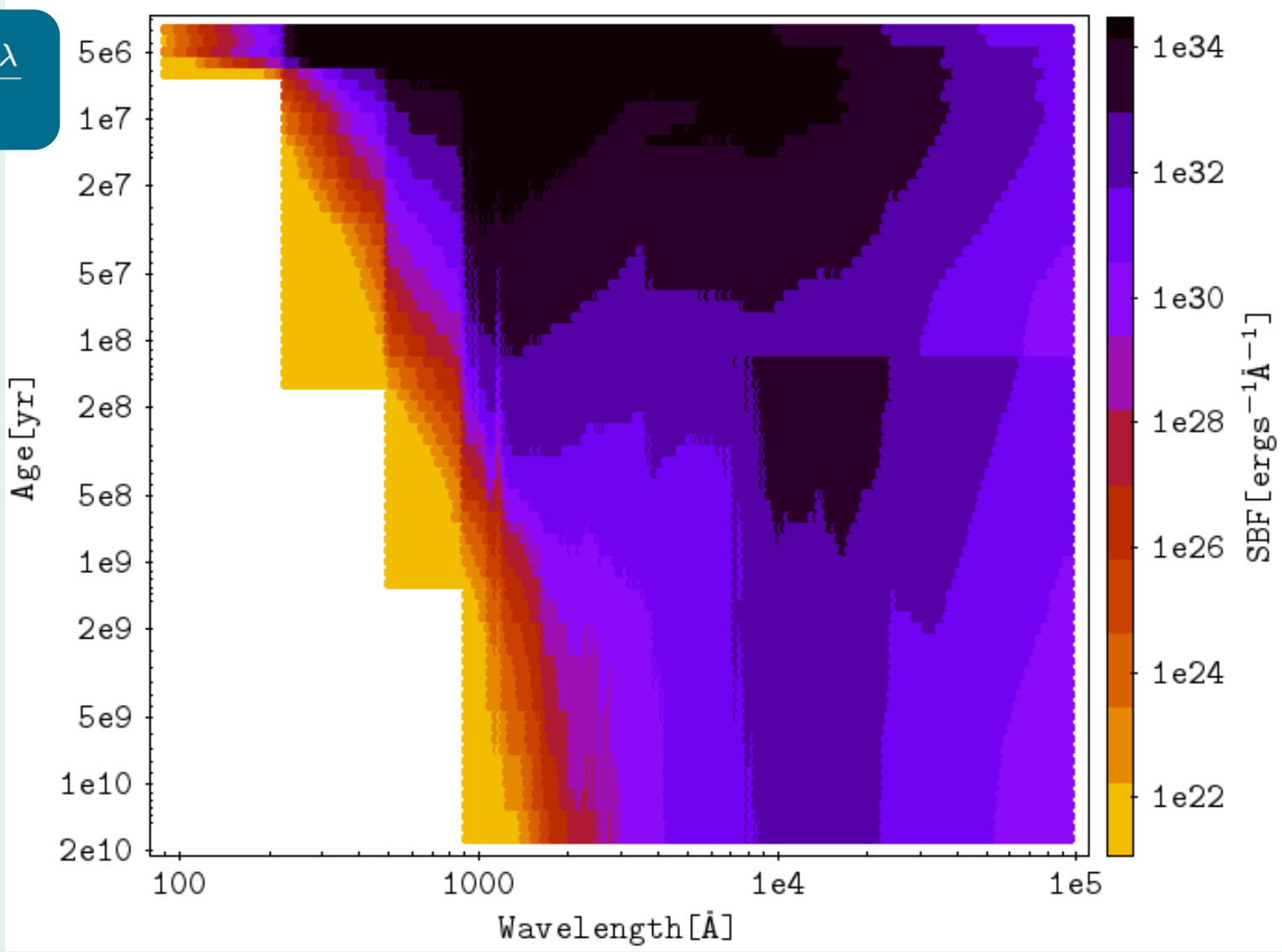
SEMI-RESOLVED STELLAR POPULATIONS AND SBF:

NEITHER FULLY INTEGRATED, NOR FULLY RESOLVED STELLAR POPULATIONS; HOW TO PLAY THIS GAME?

Miguel Cerviño

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ASOCIADO AL NASA ASTROBIOLOGY PROGRAM
CSIC

$$f_\lambda - L_\lambda \rightarrow \frac{f_\lambda - L_\lambda}{\sqrt{L_\lambda}}$$



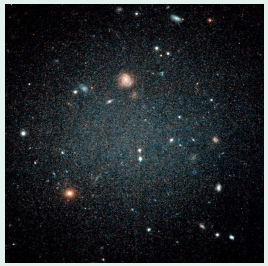
Synthesis models is the tool we use to make inferences about the ***integrated*** light of the stellar systems included in our resolution element…

In contrast CMD analysis is the method we use to make the same inferences inferences about stellar systems when we are able to see the light from ***individual*** stars on the system.

Both methods must be somehow connected, *but how is such connection?*

May you apply models to a single star?
Why "yes" or why "not"?

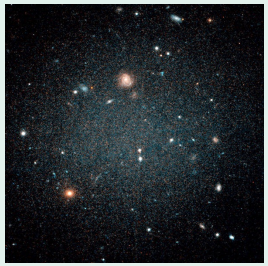
To answer this question we must understand what synthesis models do!



Let's assume that we only know the integrated light from a system. And we want to obtain all the information (in particular total masses, when and how stars was formed and so on…) from it.

The integrated (total) luminosity from a system with $\mathcal{N}$ stars is:

$$\mathscr{L}_{\lambda,\mathrm{tot}}(\mathcal{N}) = \sum_{i=1}^{\mathcal{N}} \mathscr{l}_{\lambda,i}$$



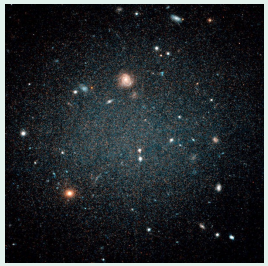
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$$\mathscr{L}_{\lambda,\mathrm{tot}}(\mathcal{N},t) = \sum_{i=1}^{\mathcal{N}} \ell_{\lambda,i} = \sum_{j=1}^{N_{\mathrm{group}(m)}} \sum_{k=1}^{N_{\mathrm{group}(t)}} w_{jk}\, \ell_{\lambda}(m_j, t-\tau_k)$$

$$\mathcal{N} = \sum_{j=1}^{N_{\mathrm{group}(m)}} \sum_{k=1}^{N_{\mathrm{group}(t)}} w_{jk}$$



The integrated luminosity from a system with $\mathcal{N}$ stars is:

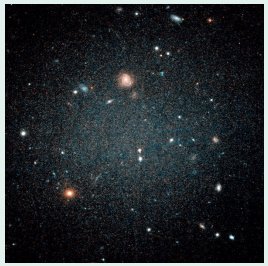
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$$\mathcal{N} = \sum_{j=1}^{N_{\mathrm{group}(m)}} \sum_{k=1}^{N_{\mathrm{group}(t)}} w_{jk}$$

$$\frac{w_{jk}}{\mathcal{N}} \xrightarrow[\mathcal{N}\to\infty]{} \mathscr{B}(m,t)$$

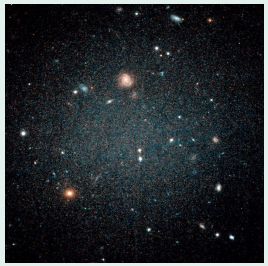
Stellar birth rate: Probability Distribution of when, with which mass (and which metallicity) would have a star born in the observed system. It includes IMF and SFR

$$\mathcal{L}_{\mathrm{tot}}[\mathcal{M}^{-1}](t) = \int_{t_0}^{t} \int_{\mathrm{m_{low}}}^{\mathrm{m_{up}}} \mathcal{B}(m,\tau)\, \ell(m,\tau)\, \mathrm{d}m\, \mathrm{d}\tau$$



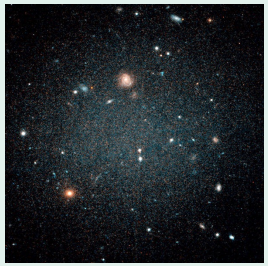
The integrated luminosity from a system with $\mathcal{N}$ stars is:

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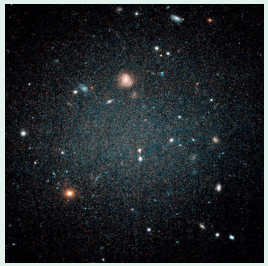
The integrated luminosity from a system with $\mathcal{N}$ stars is:

$$\mathscr{L}_{\lambda,\mathrm{tot}}(\mathcal{N},t) = \sum_{i=1}^{\mathcal{N}} \ell_{\lambda,i} = \mathcal{N} \times \left(\frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \ell_{\lambda,i} \right)$$



The integrated luminosity from a system with $\mathcal{N}$ stars is:

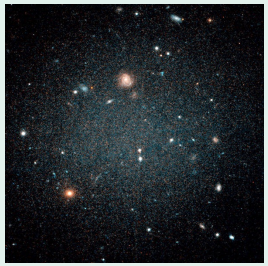
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$$\frac{\mathscr{L}_{\lambda,\mathrm{tot}}(\mathcal{N},t)}{\mathscr{M}_{\mathrm{tot}}(\mathcal{N},t)} = \frac{\sum_{i=1}^{\mathcal{N}} \ell_{\lambda,i}}{\sum_{i=1}^{\mathcal{N}} m_i} = \frac{< \tilde{\ell_\lambda}(t) >_{\mathcal{N}}}{< \tilde{m} >_{\mathcal{N}}} \xrightarrow[\mathcal{N}\to\infty]{} \frac{< \ell_\lambda(t) >_{\mathscr{B}(m,t)}}{< m >_{\mathscr{B}(m,t)}}$$



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$$\frac{\mathscr{L}_{\lambda,\mathrm{tot}}(\mathcal{N},t)}{\mathscr{M}_{\mathrm{tot}}(\mathcal{N},t)} = \frac{\sum_{i=1}^{\mathcal{N}} \ell_{\lambda,i}}{\sum_{i=1}^{\mathcal{N}} m_i} = \frac{< \tilde{\ell_\lambda}(t) >_{\mathcal{N}}}{< \tilde{m} >_{\mathcal{N}}} \xrightarrow[\mathcal{N}\to\infty]{} \frac{< \ell_\lambda(t) >_{\mathcal{B}(m,t)}}{< m >_{\mathcal{B}(m,t)}}$$

Observable

Desired Variable

Theoretical abstracción

$$\int_{t_0}^{t} \int_{\mathrm{m_{low}}}^{\mathrm{m_{up}}} \mathcal{B}(m,\tau)\, \ell(m,\tau)\, \mathrm{d}m\, \mathrm{d}\tau = \langle \ell \rangle$$

The Stellar Birth Rate is a Probability Density Function

$$\mathcal{L}_{\rm tot} = \mathcal{N} \times \left\langle \tilde{\ell} \right\rangle$$

It is the observed value and provide an estimate of the mean stellar luminosity

$$\langle \mathcal{L}_{\rm tot} \rangle = \mathcal{N} \, \langle \ell \rangle$$

It is the theoretical value, which is and abstraction (and exactly coincided with the observed one for infinite stars)

$$\left\langle \tilde{\ell} \right\rangle \neq \langle \ell \rangle \qquad\qquad \mathcal{L}_{\rm tot} \neq \langle \mathcal{L}_{\rm tot} \rangle$$

$\mathcal{L}_{\rm tot}$ is distributed around $\langle \mathcal{L}_{\rm tot} \rangle$

$\left\langle \tilde{\ell} \right\rangle$ is distributed around $\langle \ell \rangle$

If there is a distribución, it must be a variance (i.e. square of the standard deviation, $\sigma^2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})]$) of the posible (i.e. observed) $\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})$ values. Actually, it can be shown that (Cerviño & Luridiana 2006):

$$\sigma^2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \sigma^2(\ell_\lambda)$$

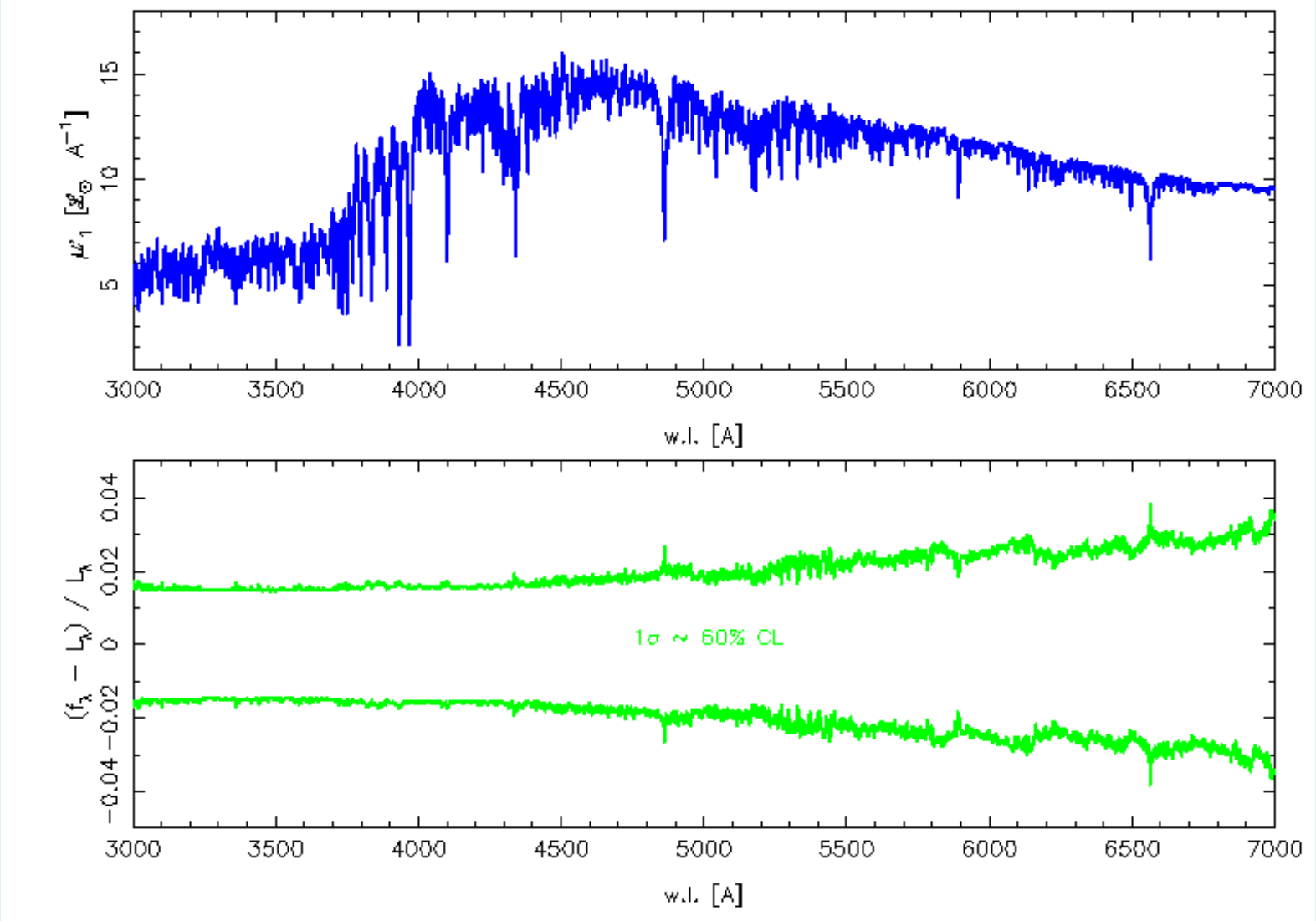
Note that the variance increase with the number of stars, but the relative dispersion decreases:

$$\frac{\sigma[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})]}{\langle \mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N}) \rangle} = \frac{1}{\sqrt{\mathscr{N}}} \frac{\sigma(\ell_\lambda)}{\langle \ell_\lambda \rangle} \xrightarrow[\mathscr{N}\to\infty]{} 0$$

Note that a gaussian is defined by just its mean and its variance and that χ^2 fitting assumes intrinsically gaussian-like underling distributions

Note that the *theoretical dispersion* increases in the IR

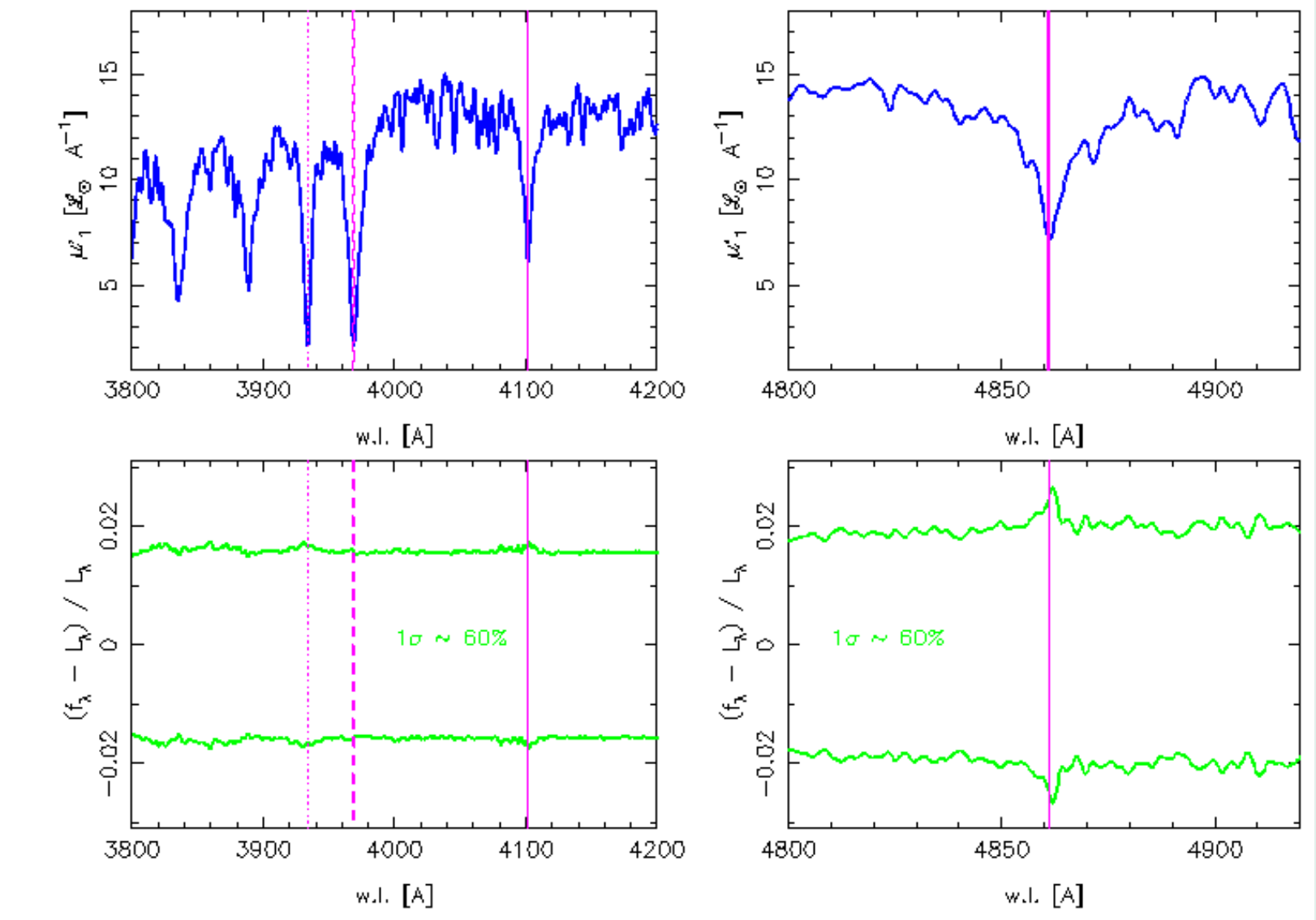
Not all wavelengths must have the same impacta in the fit!!



1 Gyr, solar metallicity 10^{5}* cluster

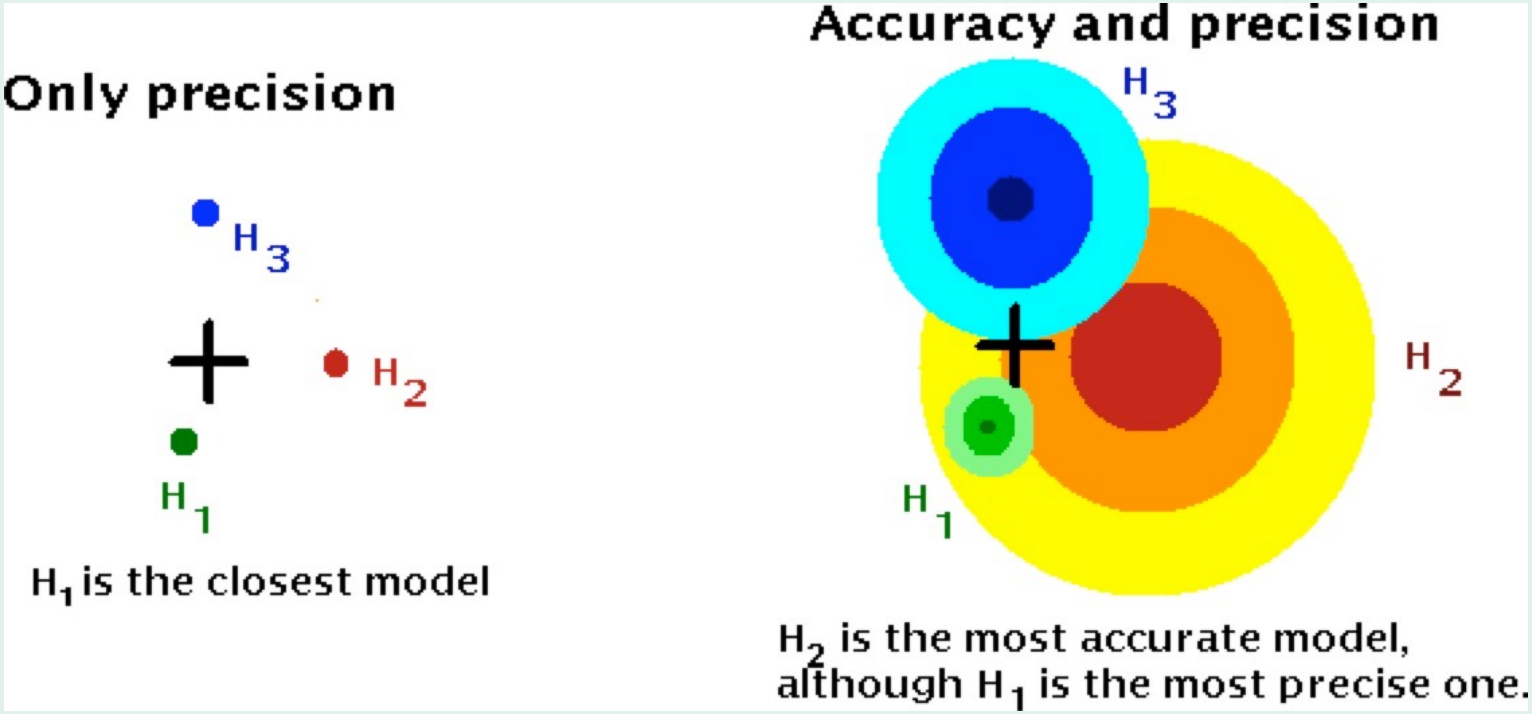
Note that the *theoretical dispersion* depends the line!!

Actually Balmer lines have intrinsically a larger variance, so do not force this region



1 Gyr, solar metallicity 10^{5}* cluster

In general, the effect is more relevant if the data have an error lower than the theoretical dispersion of the model.



It would be specially relevant for IFU data in close galaxies!! (or for ELT-like observations)

Note: Actually there is also correlations between different wavelenghts which must also be taken into account.

Tonry & Schneider, defined SBF as the ratio of the variance over the mean value of the stellar luminosity distribution.

Note that they explain the origin of SBF as Poisson statistics in the number of stars, but Poisson statistics only provide 0.1% of the fluctuation. The fluctuación is *due to the different types of stars in the population*, and not due Poison stastics of a particular type or evolutionary phase (Cerviño, Luridiana & Jamet 2008). *SBF is a global property of the population, just like* $\left\langle \mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N}) \right\rangle$ *or* $\left\langle \ell_\lambda \right\rangle$

$$\sigma^2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \sigma^2(\ell_\lambda)$$

$$\left\langle \mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N}) \right\rangle = \mathscr{N} \times \left\langle \ell_\lambda \right\rangle$$

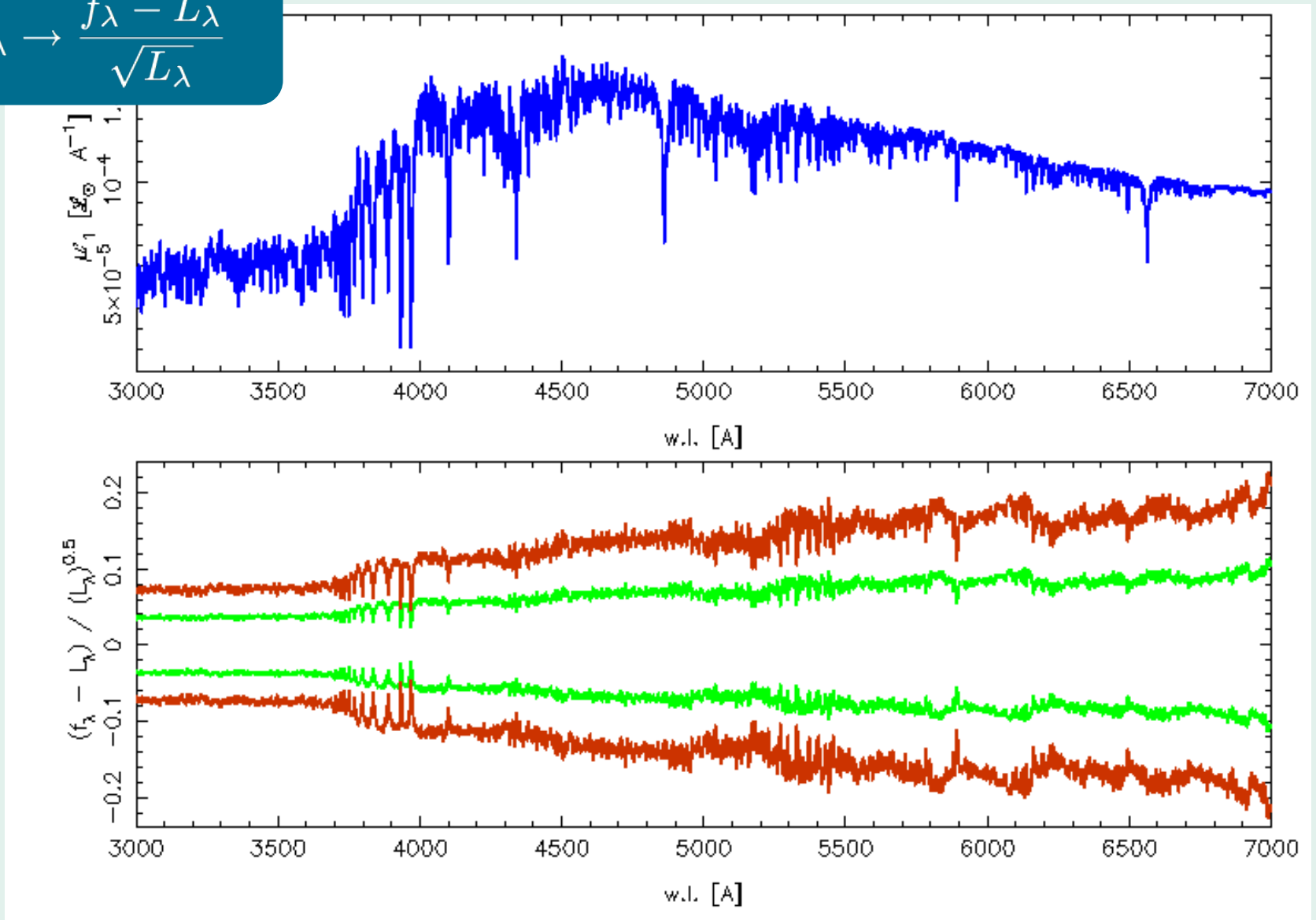
So:

$$\bar{\mathscr{L}}_{\lambda,\mathrm{tot}}(\mathscr{N}) = \frac{\sigma^2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})]}{\left\langle \mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N}) \right\rangle} = \frac{\sigma^2(\ell_\lambda)}{\left\langle \ell_\lambda \right\rangle} = \bar{\ell}_\lambda$$

See Tonry & Schneider 1988, AJ 93, 807;

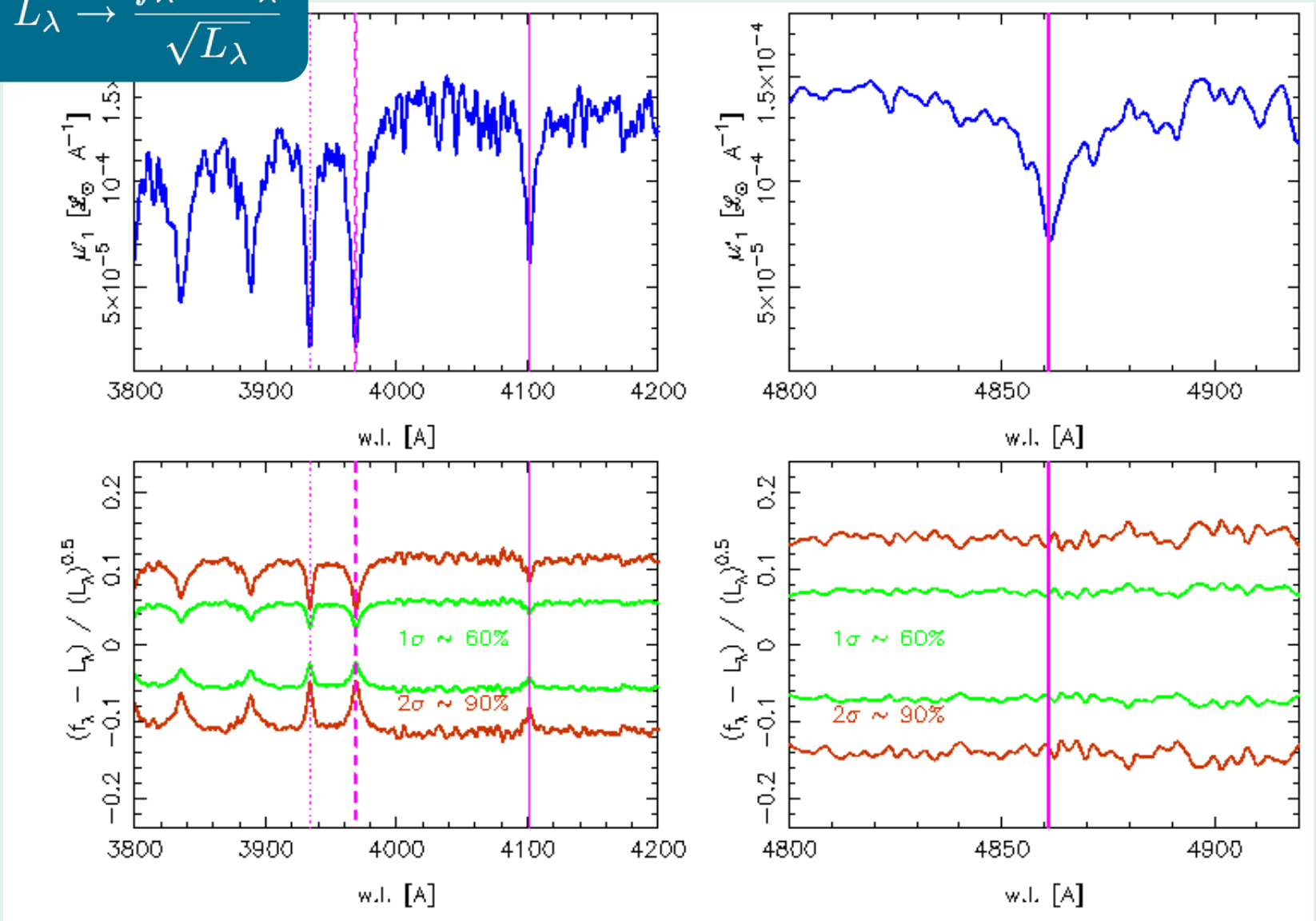
Cerviño, Luridiana & Jamet 2008, A&A 491,693

$$f_\lambda - L_\lambda \rightarrow \frac{f_\lambda - L_\lambda}{\sqrt{L_\lambda}}$$



1 Gyr, solar metallicity 10^{5*} cluster

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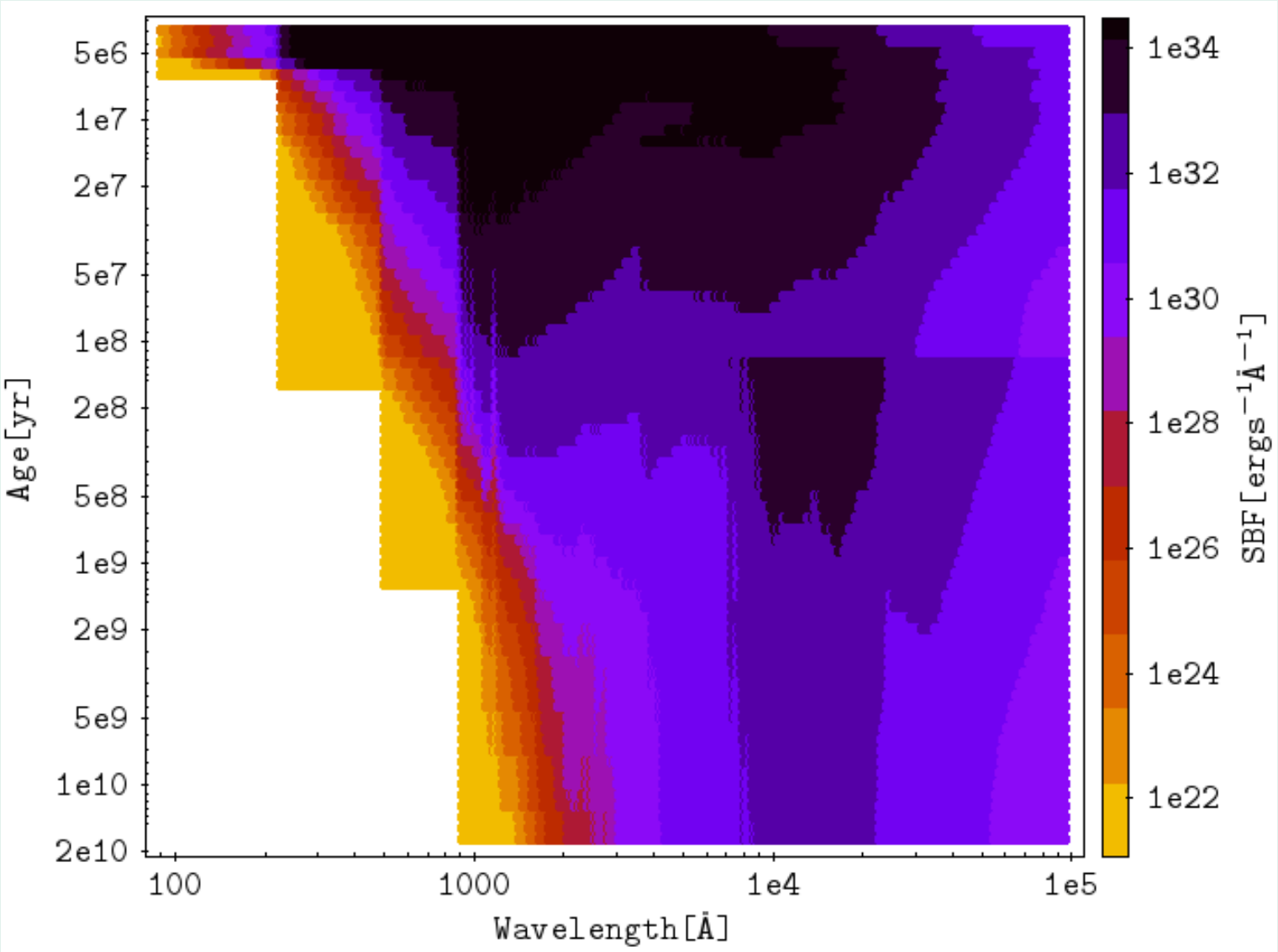


Fig. from SED@ models server (Cerviño in prep; also from data of Gonzalez Delgado et al. 2005 models); see also Vazdekis et al. 2002 MNRAS 493, 5131, Rodriguez-Beltrán et al. MNRAS 2021, 507, 3005

However, once we realize that SP models results are distributions, the variance is not the whole history…

$$\frac{w_{jk}}{\mathcal{N}} \xrightarrow[\mathcal{N}\to\infty]{} \mathcal{B}(m,t) \qquad \int_{t_0}^{t}\int_{\mathrm{m_{low}}}^{\mathrm{m_{up}}} \mathcal{B}(m,\tau)\,\ell(m,\tau)\,\mathrm{d}m\,\mathrm{d}\tau = \langle\ell\rangle$$

What is the implication of $\xrightarrow[\mathcal{N}\to\infty]{}$?

Or much better:

how varies the distribution of $\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N},t)$ with $\mathcal{N}$?

Is gaussianity (and so the power of χ^2 fitting) lost?

Cerviño & Luridiana 2006, A&A 451, 475 showed that it is possible obtain the parameters of the $\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})$ distribution as:

$$\mu'_1[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \mu'_1(\ell_\lambda)$$

$$\kappa_n[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \kappa_n(\ell_\lambda) \quad \forall n$$

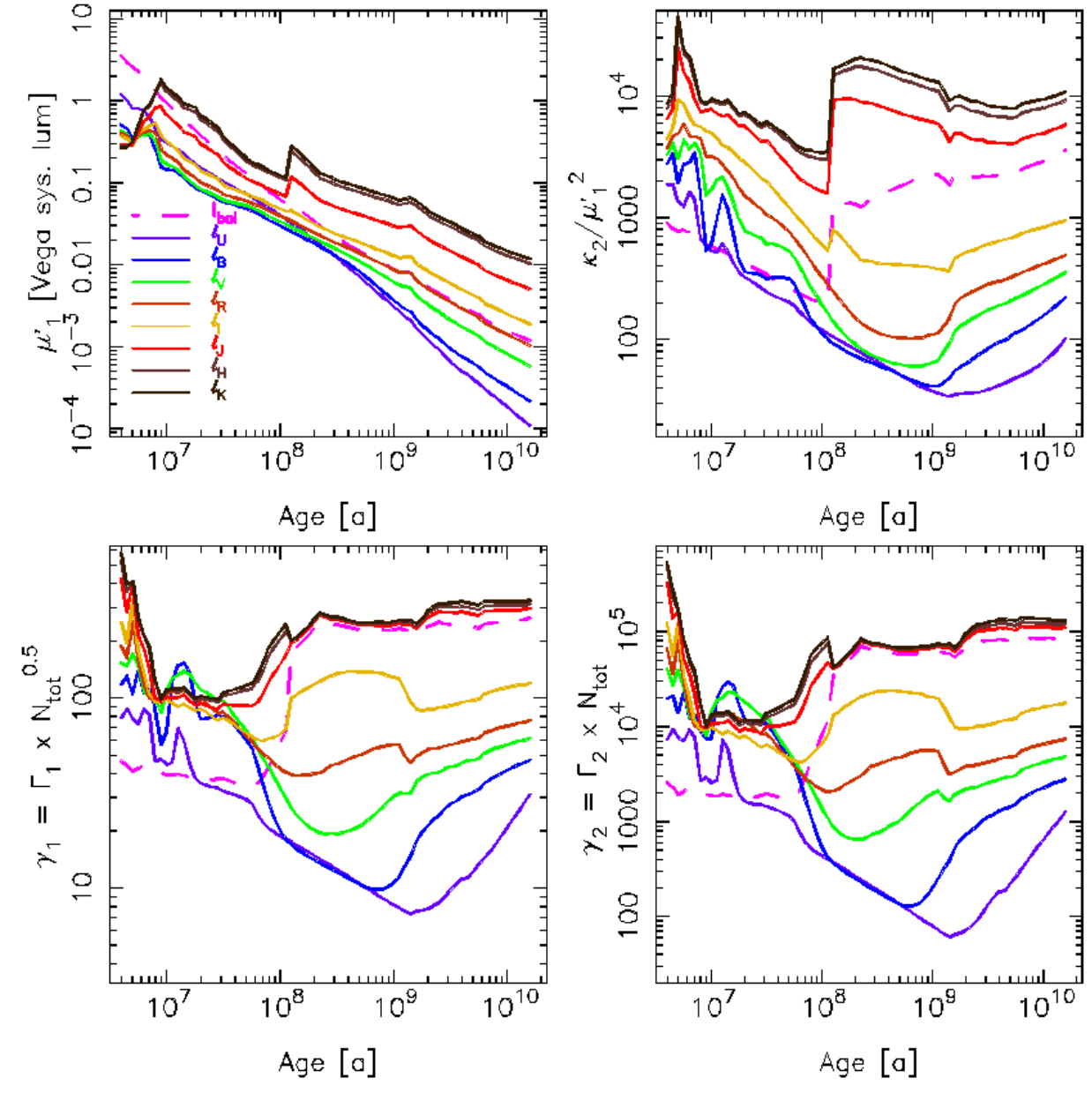
With $\kappa_2 = \sigma^2$ (the variance) and the skewness (asymmetry) and Kurtosis defined as:

$$\gamma_1 = \frac{\kappa_3}{\sqrt{\kappa_2^3}} \qquad \gamma_2 = \frac{\kappa_4}{\sqrt{\kappa_2^4}} = \frac{\kappa_4}{\kappa_2^2}$$

Reasonable skewness and kurtosis for χ^2 fitting (gaussian distributions) must be below 0.01

$$\gamma_1[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \frac{1}{\sqrt{\mathscr{N}}}\gamma_1(\ell_\lambda)$$

$$\gamma_2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \frac{1}{\mathscr{N}}\gamma_2(\ell_\lambda)$$



Source: Cerviño & Luridiana 2006, A&A 451, 475

Case of N= 1:

Individual stars

The possible luminosities are defined in in the HR diagram

In Fig. Case of all stars born at the same time
The color is the density, given by $\mathcal{B}(m,t)$ by a instantaneous burst of 1 Myr

The distribution of stars for given $\mathcal{B}(m,t)$ is given by:

$$\mu_1'(\ell_\lambda) = \langle \ell_\lambda \rangle$$

$$\kappa_n(\ell_\lambda)$$

$$\gamma_1(\ell_\lambda) = \frac{\kappa_3(\ell_\lambda)}{\kappa_2(\ell_\lambda)^{3/2}}$$

$$\gamma_2(\ell_\lambda) = \frac{\kappa_4(\ell_\lambda)}{\kappa_2^2(\ell_\lambda)}$$

See Cerviño & Luridiana 2006, A&A 451, 475

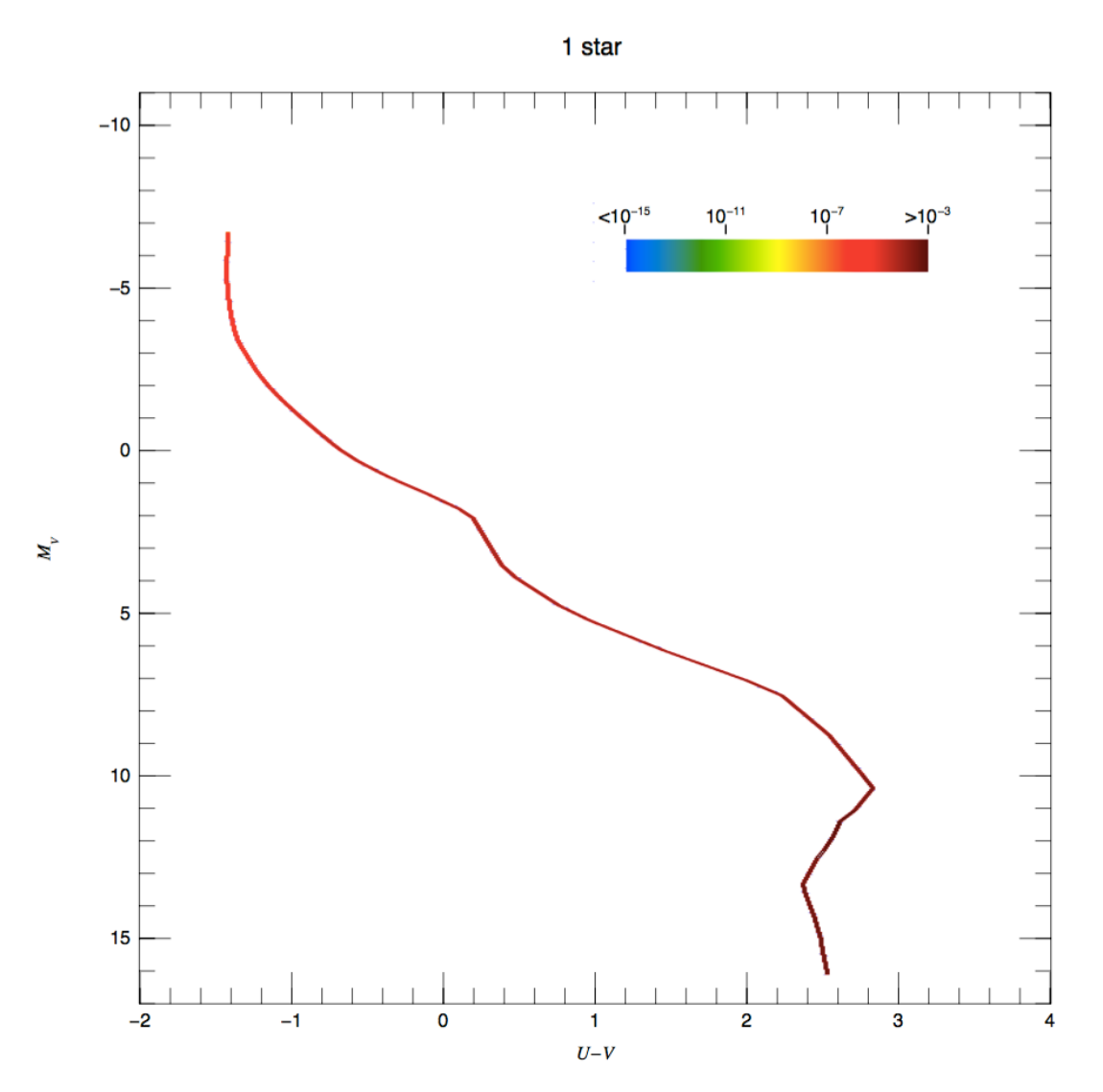


Fig Source: Maíz Apellániz, J., 2008. ApJ 677, 1278.

Case of $\mathcal{N}$= 2:

Each point contains 2 stars

In Fig. Case of all stars born at the same time with the same $\mathcal{B}(m,t)$ as in previous figure

The distribution of stars, when taken 2 by 2 for given $\mathcal{B}(m,t)$ is given by:

$$\mu_1'[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2)] = 2 \times \mu_1'(\ell_\lambda)$$

$$\kappa_n[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2)] = 2 \times \kappa_n(\ell_\lambda) \quad \forall n$$

$$\gamma_1[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2)] = \frac{1}{\sqrt{2}}\gamma_1(\ell_\lambda)$$

$$\gamma_2[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2)] = \frac{1}{2}\gamma_2(\ell_\lambda)$$

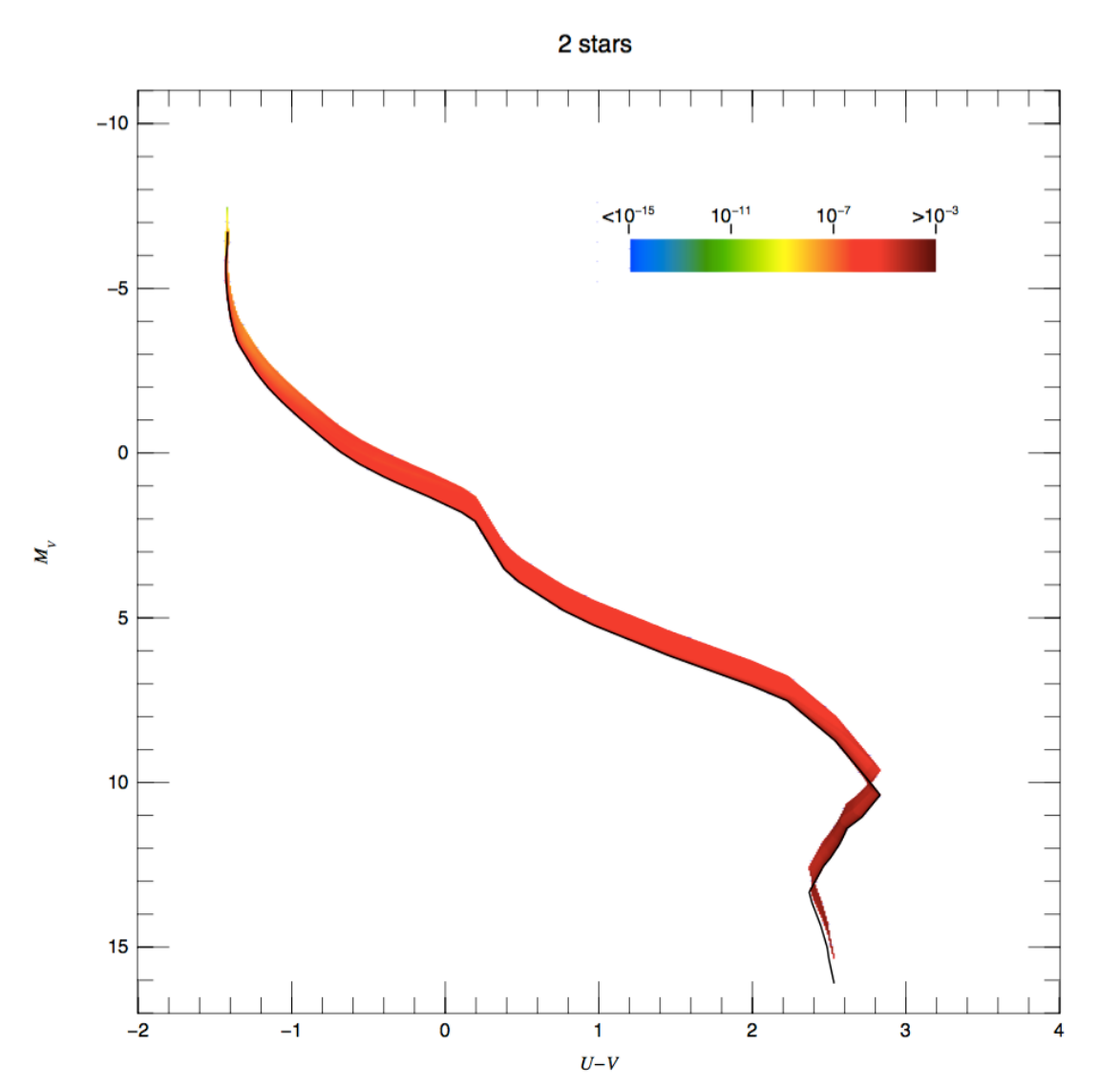


Fig Source: Maíz Apellániz, J., 2008. ApJ 677, 1278.

See Cerviño & Luridiana 2006, A&A 451, 475

Case of $\mathcal{N}$= 64:

Each point contains 64 stars

In Fig. Case of all stars born at the same time with the same $\mathcal{B}(m,t)$ as in previous figure

The distribution of stars, when taken 64 by 64, for given $\mathcal{B}(m,t)$ is given by:

$$\mu_1'[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=64)] = 64 \times \mu_1'(\ell_\lambda)$$

$$\kappa_n[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=64)] = 64 \times \kappa_n(\ell_\lambda) \quad \forall n$$

$$\gamma_1[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=64)] = \frac{1}{\sqrt{64}}\gamma_1(\ell_\lambda)$$

$$\gamma_2[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=64)] = \frac{1}{64}\gamma_2(\ell_\lambda)$$

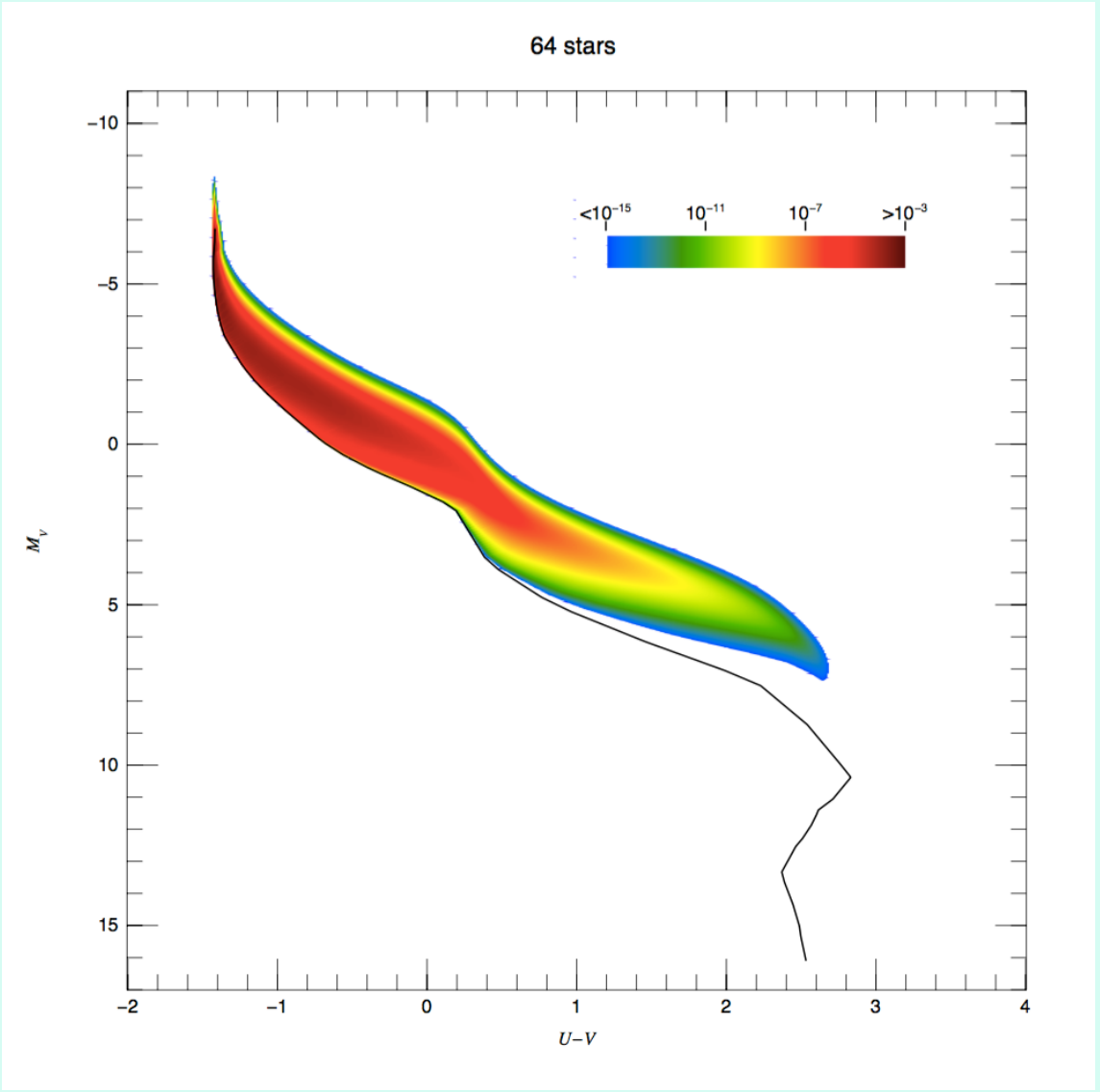


Fig Source: Maíz Apellániz, J., 2008. ApJ 677, 1278.

See Cerviño & Luridiana 2006, A&A 451, 475

Case of $\mathcal{N}$= 2048:

Each point contains 2048 stars

In Fig. Case of all stars born at the same time with the same $\mathcal{B}(m,t)$ as in previous figure

The distribution of stars, when taken 2048 by 2048, for the given $\mathcal{B}(m,t)$:

$$\mu_1'[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2048)] = 2048 \times \mu_1'(\ell_\lambda)$$

$$\kappa_n[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2048)] = 2048 \times \kappa_n(\ell_\lambda) \quad \forall n$$

$$\gamma_1[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2048)] = \frac{1}{\sqrt{2048}}\gamma_1(\ell_\lambda)$$

$$\gamma_2[\mathcal{L}_{\lambda,\mathrm{tot}}(\mathcal{N}=2048)] = \frac{1}{2048}\gamma_2(\ell_\lambda)$$

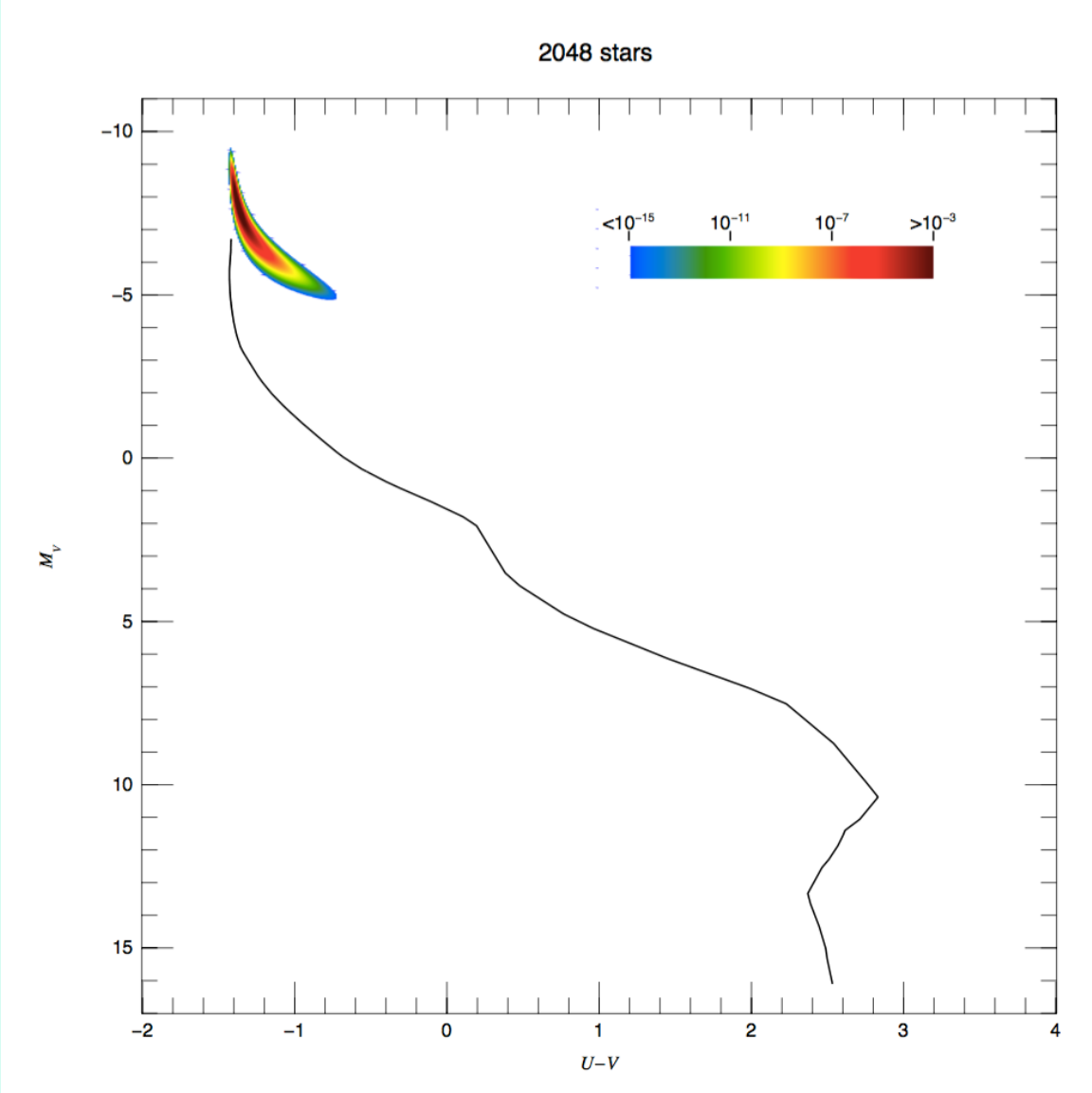
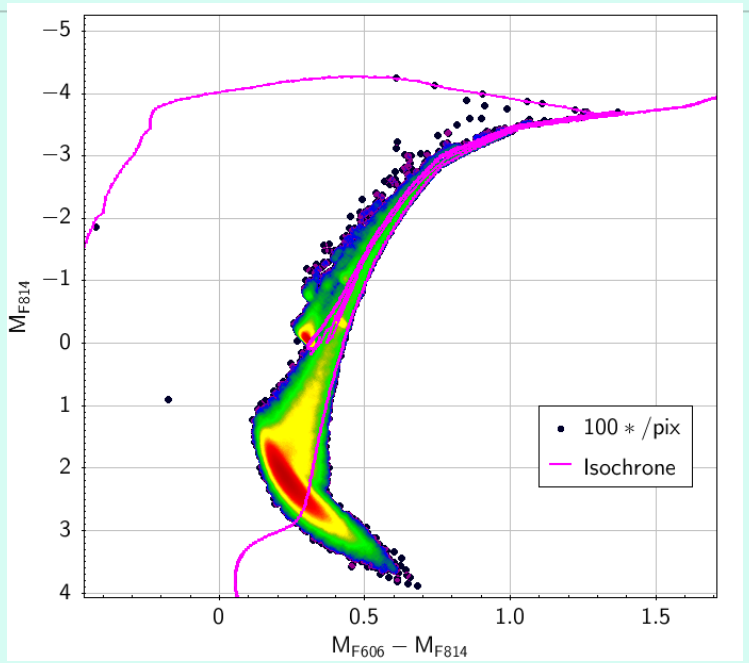
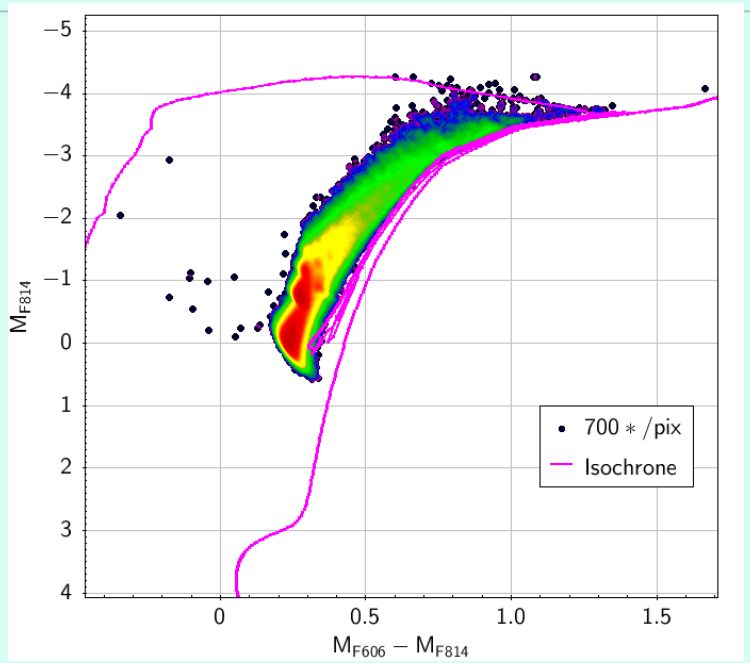
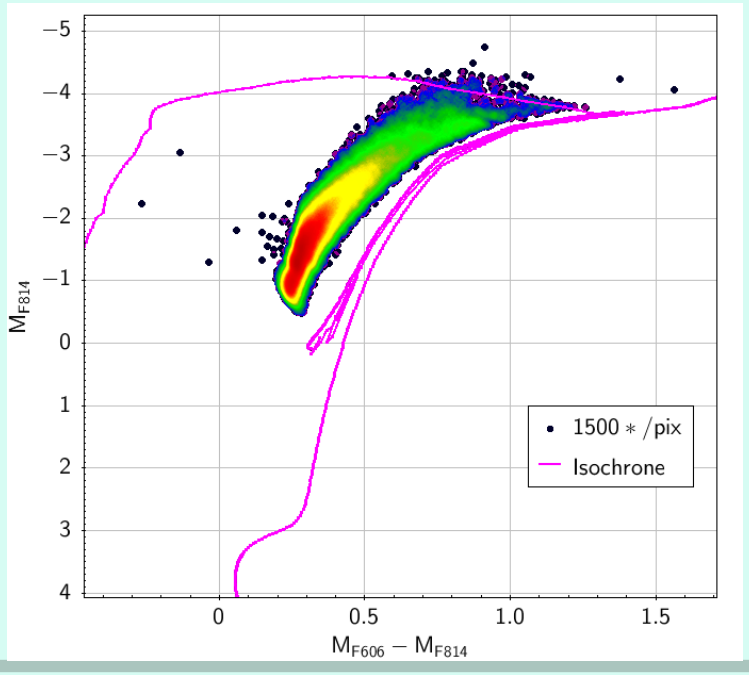


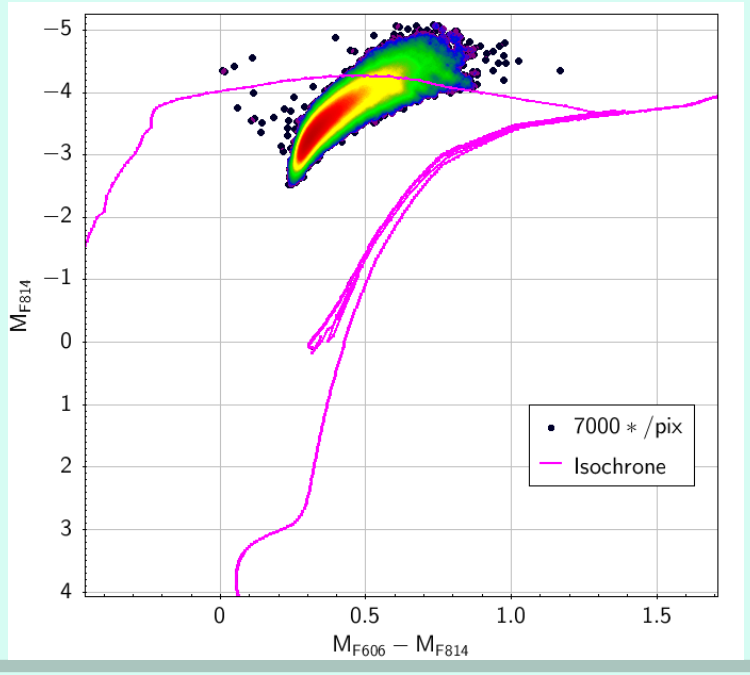
Fig Source: Maíz Apellániz, J., 2008. ApJ 677, 1278.

See Cerviño & Luridiana 2006, A&A 451, 475









160000 MC simulations of an IB at 10 Gyr with different number of stars per pixel: 100, 700, 1500 and 7000 */ pixel)

In (average) total mass in each pixel is: 31.5, 220.5, 472.5 and 2205 Mo

Kroupa IMF [0.1-1 Mo]

$< m > = 0.315$

Metallicity: Z= 0.001

In general

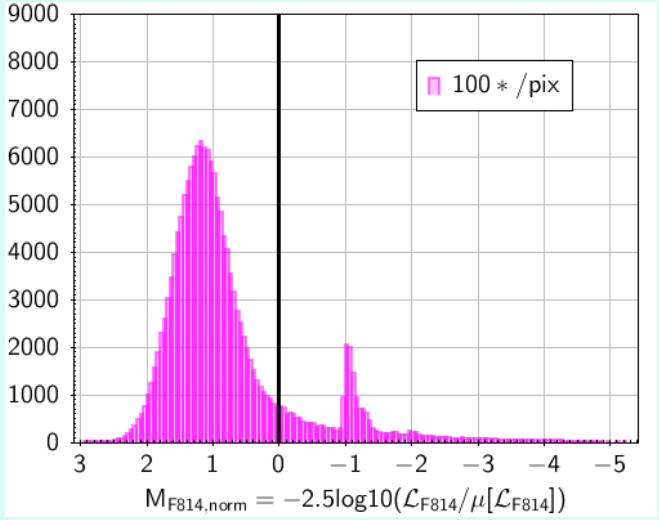
The distribution of observed properties have an spread WHICH CAN BE MODELED depending on number of stars INCLUDED in the observation (Cerviño & Luridiana 2006):

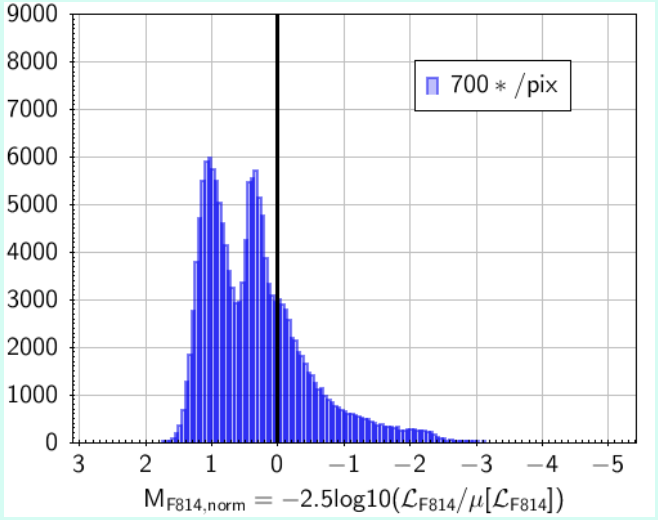
$$\mu_1'[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \mu_1'(\ell_\lambda)$$

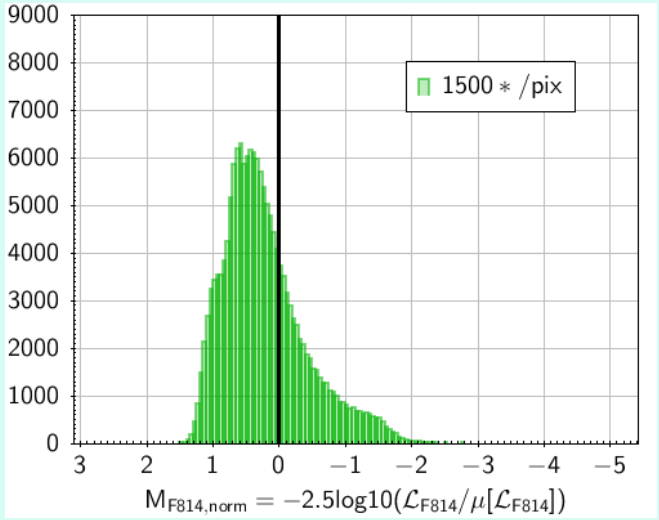
$$\kappa_n[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \mathscr{N} \times \kappa_n(\ell_\lambda) \quad \forall n$$

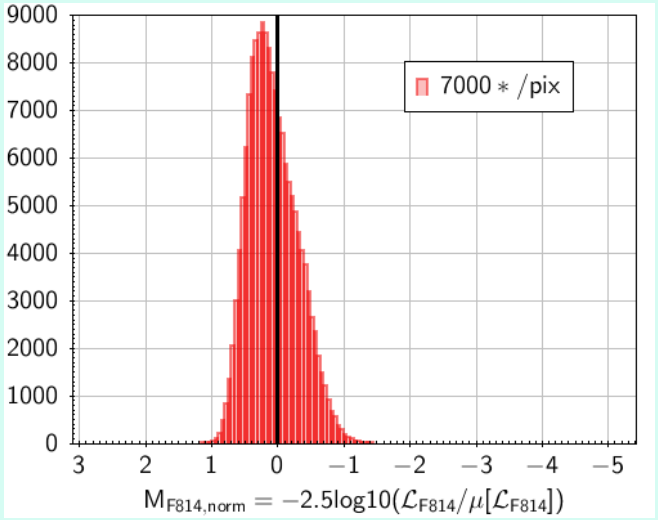
$$\gamma_1[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \frac{1}{\sqrt{\mathscr{N}}}\gamma_1(\ell_\lambda)$$

$$\gamma_2[\mathscr{L}_{\lambda,\mathrm{tot}}(\mathscr{N})] = \frac{1}{\mathscr{N}}\gamma_2(\ell_\lambda)$$

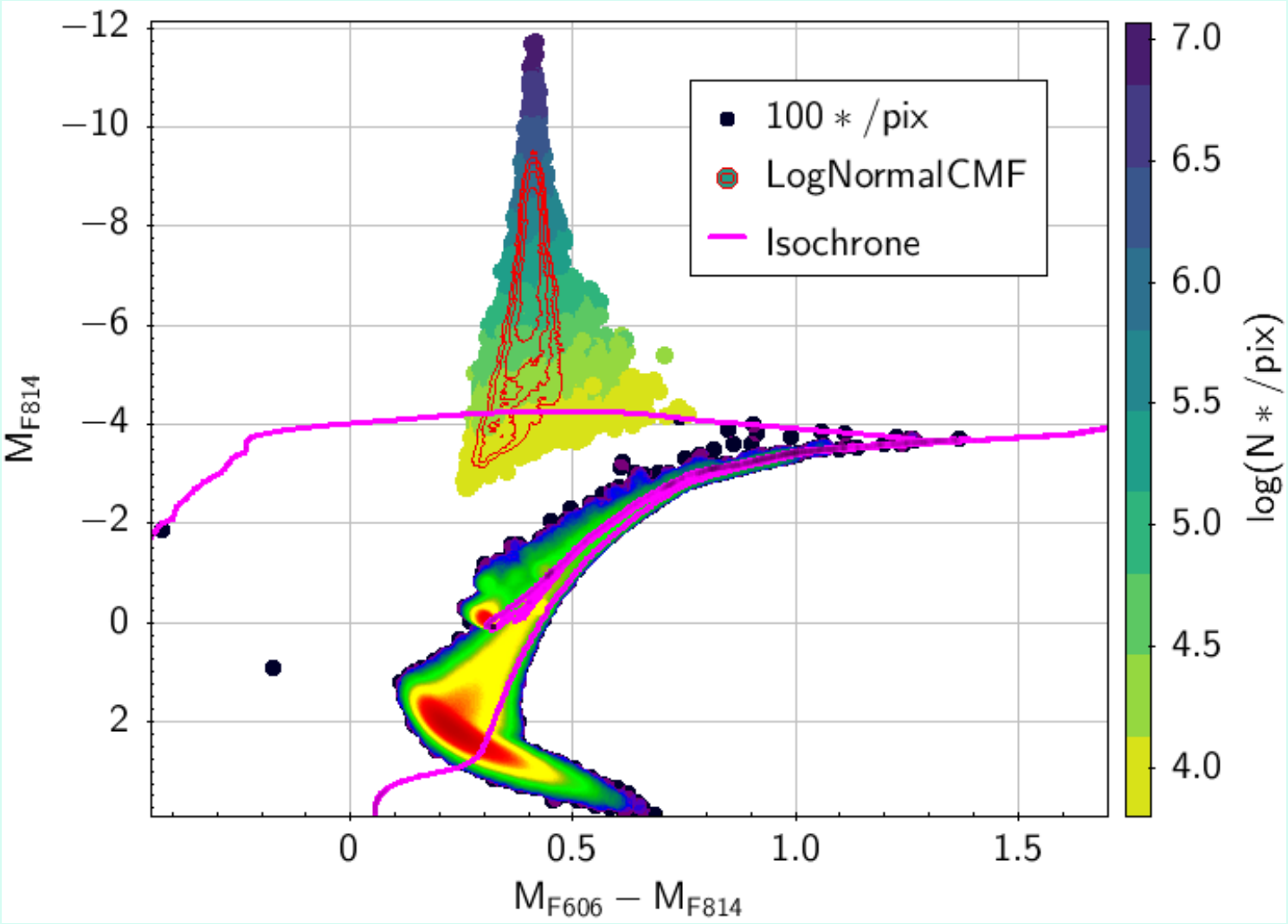








See Cerviño & Luridiana 2006, A&A 451, 475

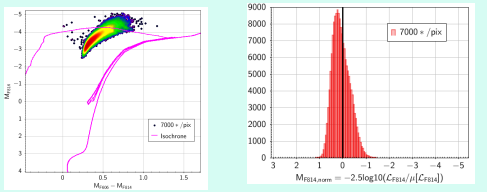


160000 MC simulations of an IB at 10 Gyr with 100 stars per pixel

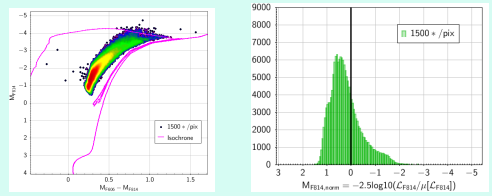
+

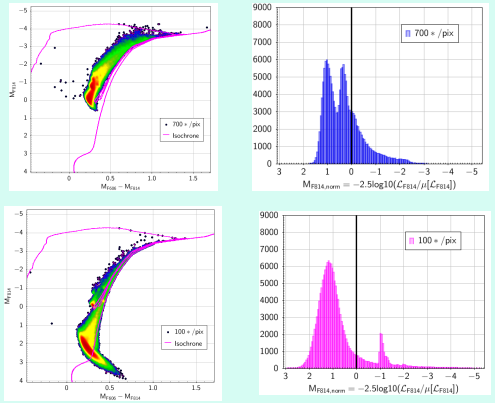
10000 MC simulations of an IB at 10 Gyr with a number of stars per pixel following a lognormal distribution

$N_{pix} = 1$; $n_{*/pix} = \mathcal{N}_{tot}$



$N_{pix} = n$; $n_{*/pix} = \mathcal{N}_{tot}/n$





Spatial resolution increases n*/ pix decreases

$N_{pix} = \mathcal{N}_{tot}$; $n_{*/pix} = 1$ (~CMD)

pix = IFU = spaxel

There is a trade-off between:

- how many resolution elements you have N_{pix} (or m_{pix})
- how many stars are in each resolution element $n_{*/pix}$ (or $m_{*/pix}$)

A physical constraint: For any given system there is a finite number of stars:

$$\mathcal{N}_{tot} = N_{pix} \times n_{*/pix} \text{ is fixed}$$

we must take into account the underlying probability distribution for a given $n_{*/pix}$!! (which can be obtained theoretically)

and extreme care for non gaussian distributions!!

See Cerviño 2013, NewAstronomy Reviews 57, 12 for a review in the subject

Minimum mass required for

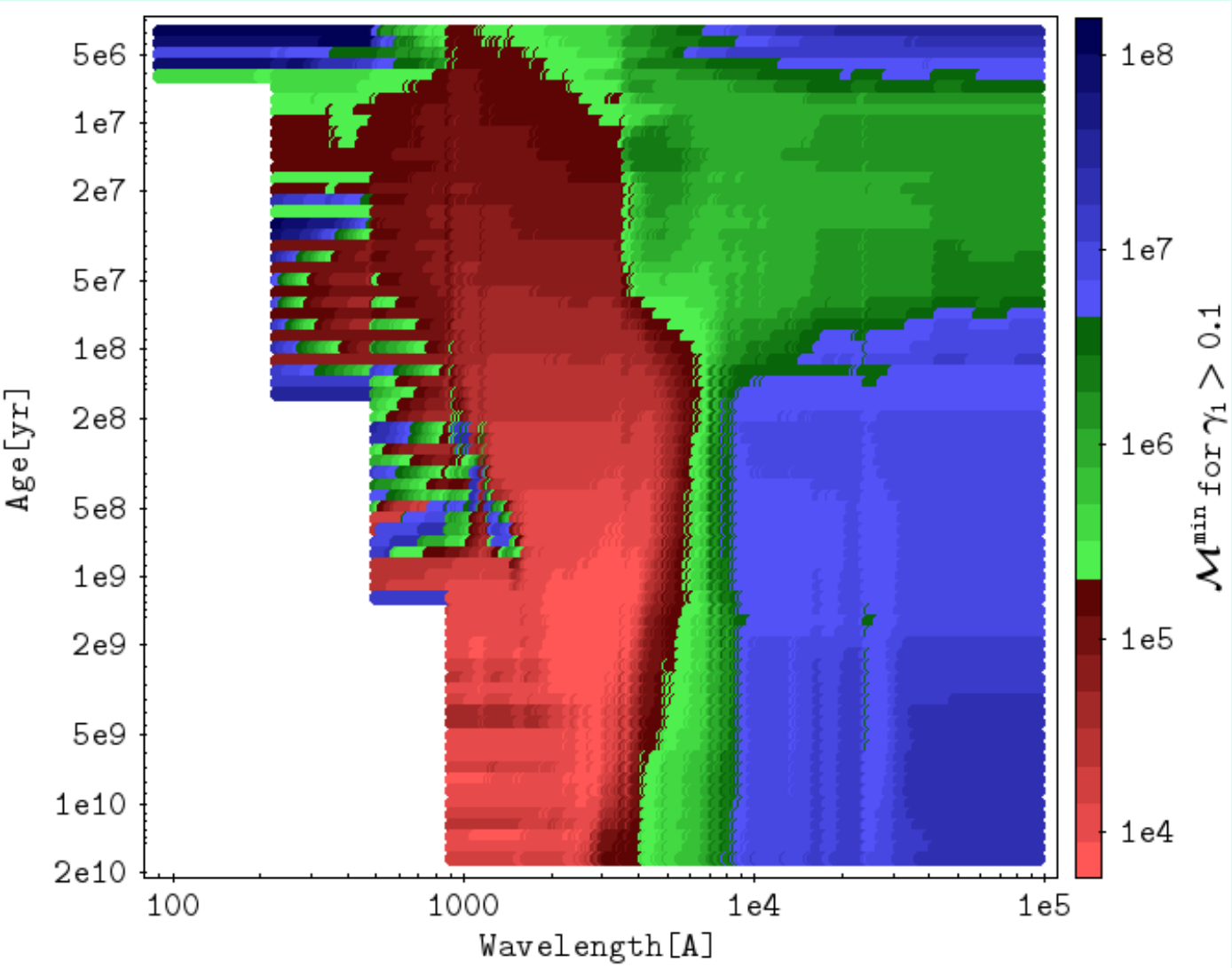
$$\gamma_1 < 0.1$$

as a function of age and wavelength

Skewness and kurtosis roughly vanish when the system contains at least 10^5 stars (10^7 for IR and ionizing flux)

Only mean a nd variance remains (hence gaussian-like distributions)..

… hence a metric of fitting remains (Visible is MUST BE better fitted than IR)



See Cerviño 2013, NewAstronomy Reviews 57, 12 for a review in the subject

A rule of thumb: If your "cluster" luminosity is similar to the luminosity of an individual star you the distribution is not gaussian for sure!!

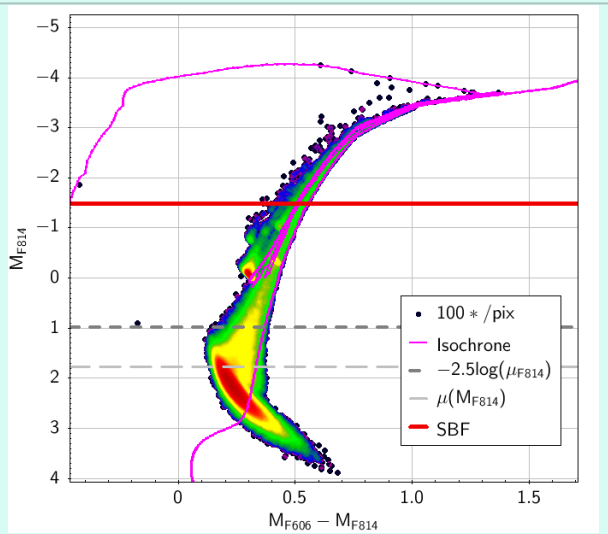
Calibration of the SBF

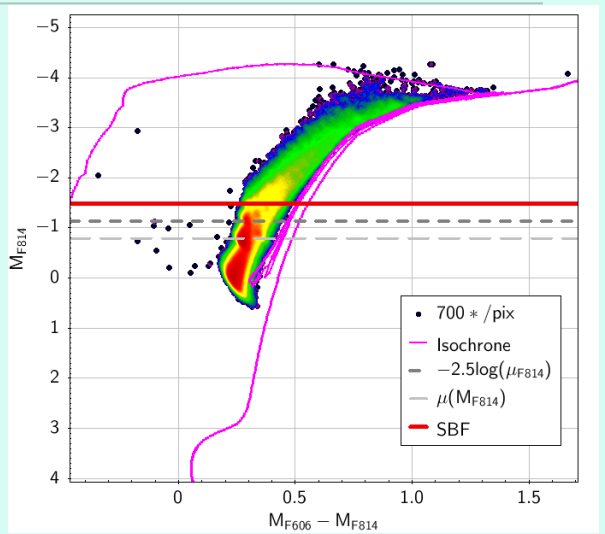
Note: The calibration is done by observations of systems with known distance
Such system are “bona fide” system, it is massive elliptical galaxies with a large number of stars per pixel

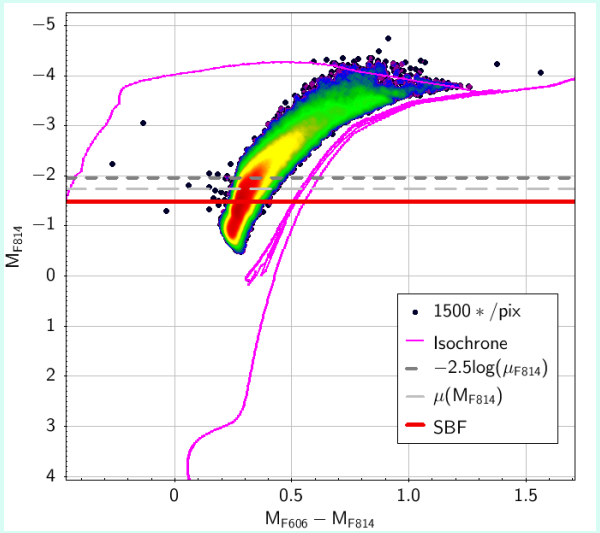
Actually the method is calibrated with systems where $n_{*/pix}$ is large enough to have tens of red giants stars (the one “associated” with the fluctuation), as proposed by Tonry)

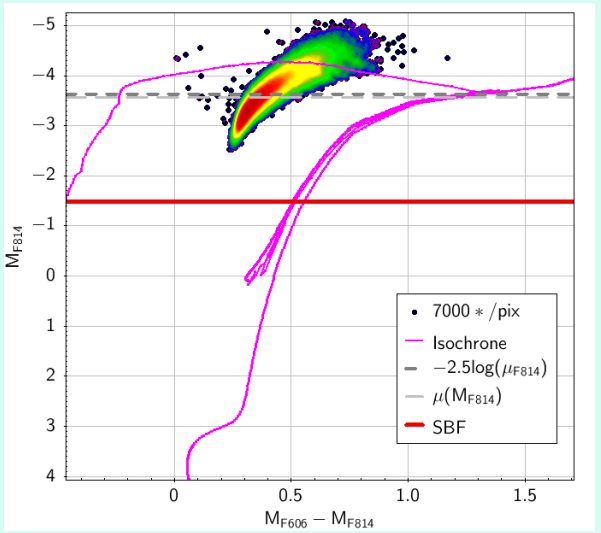
It means, the method assume, $n_{*/pix}$ so the mean value of luminosity per pixel is about 10 times the associated SBF i.e. at least 2.5 mag higher than the SBF

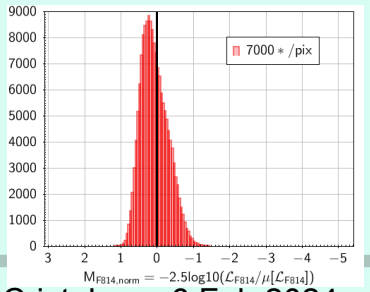
⇒ *In probability*: the distribution of mean values per pixel (and, so, fluctuations) are nearby gaussian

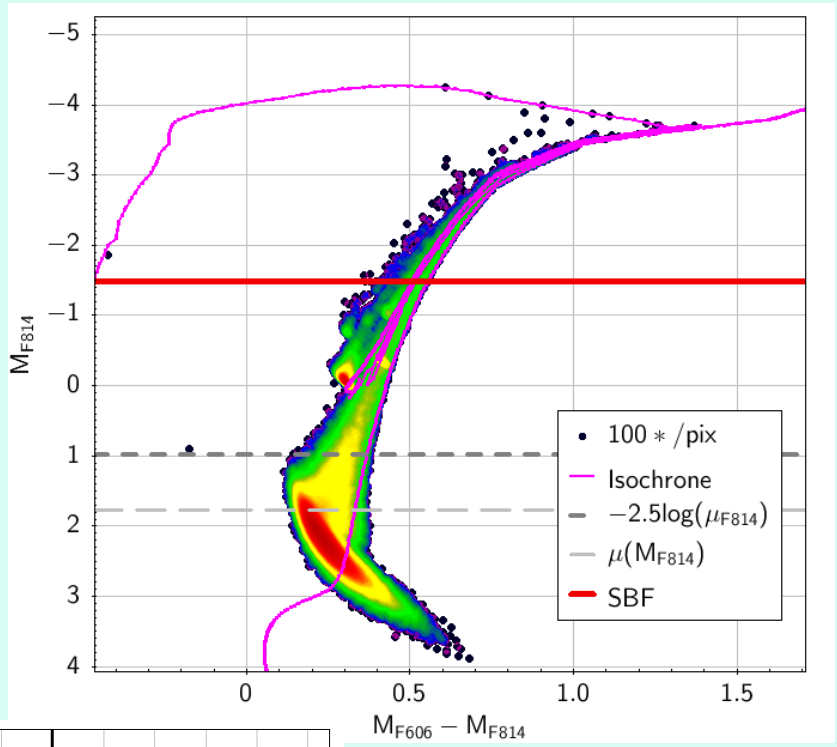


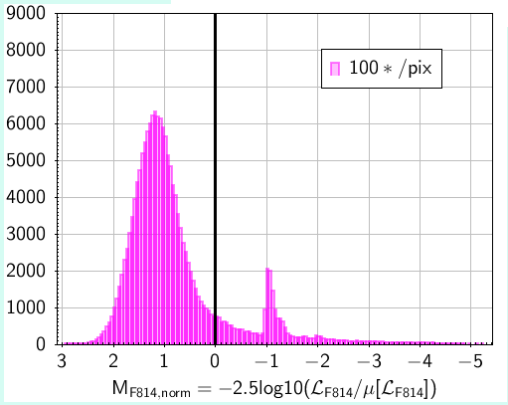




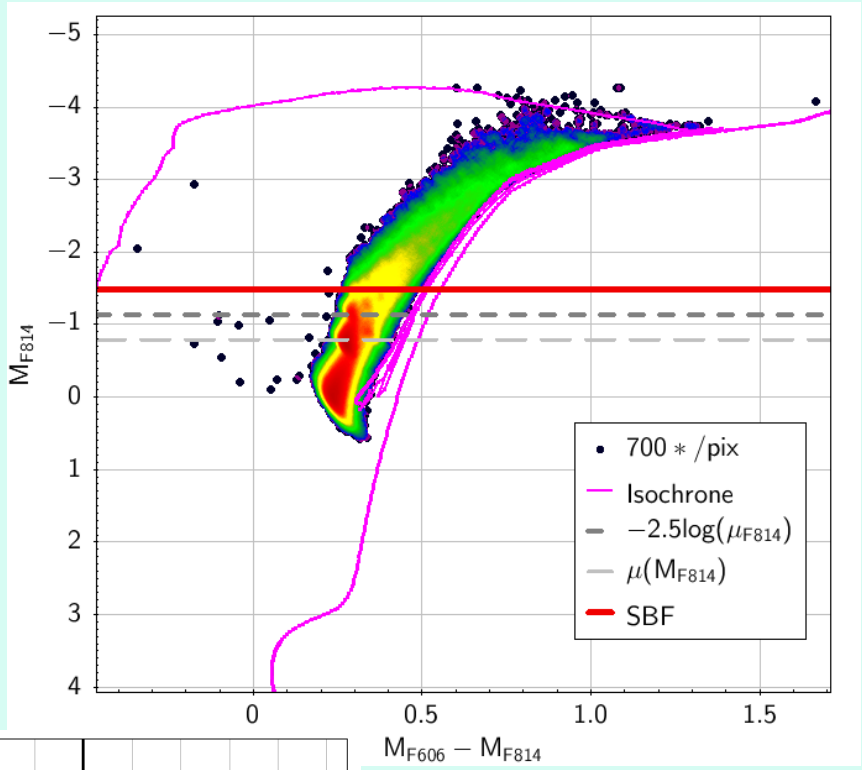


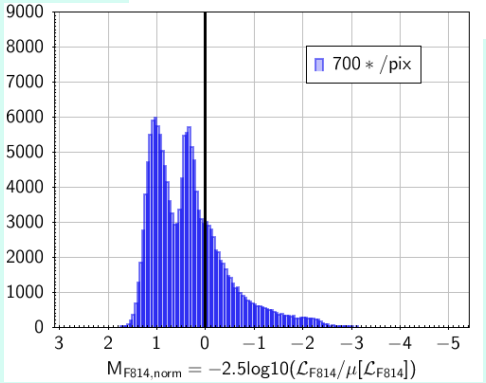






Possible problems in obtain the local mean needed for SBFs!!

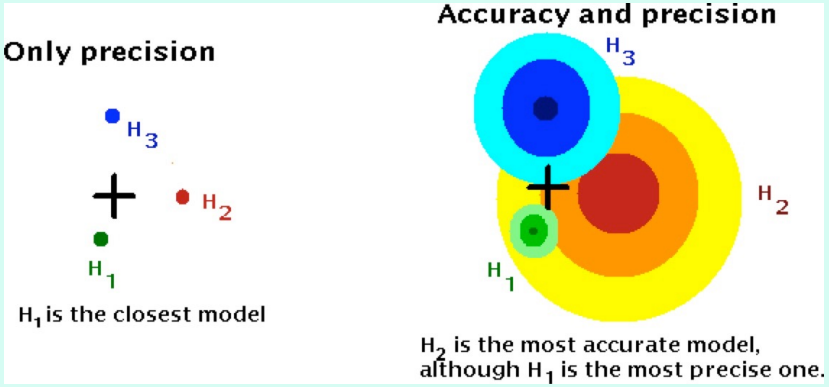




Additional bias in the mean estimate due to the noise/ background!!

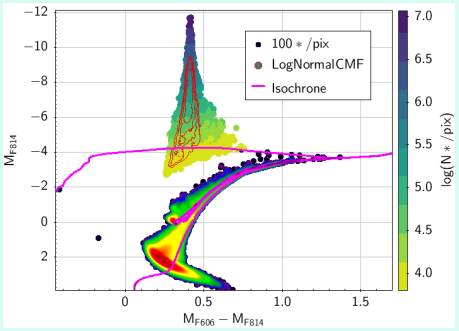
I.- Characterize model "uncertainty" is relevant, since it is an observable (as *SBFs measurements show*).

II.- If we are in a gaussian regime, the intrinsic metric of fitting due to sampling effects can be modeled by χ^2 statistics. However, for full spectral fitting not al wavelengths must fit equally good (residuals in IR must be larger!!)



III.- In general, do not assume gaussian distributions for granted, verify before if gaussianity applies !! (when $\xrightarrow[\mathcal{N}\to\infty]{}$ apply)

Always try to estimate how many stars (or mass) are in your *resolution element*



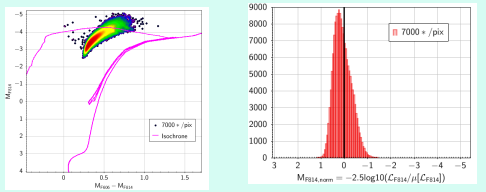
IV.- When spatial resolution increases n*/pix decreases. In such cases, you must think as in a CMD: The information is not in each individual star (pixel in your case), but in the distribution of the ensemble of stars (or pixels).
It is the shape of the distribution who have the real information (not the individual pixels/IF/spaxel)

So:

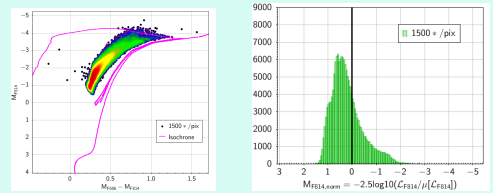
May you apply models to a single star?

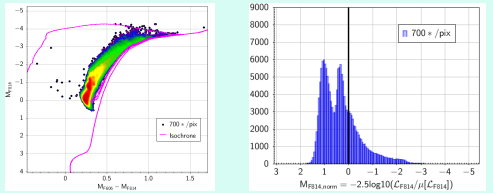
Yes, but you must take into account that modes are more than just the usual mean value

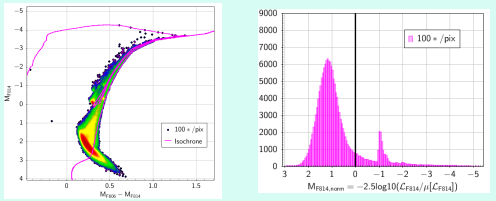
$N_{pix} = 1$; $n_{*/pix} = \mathcal{N}_{tot}$



$N_{pix} = n$; $n_{*/pix} = \mathcal{N}_{tot}/n$

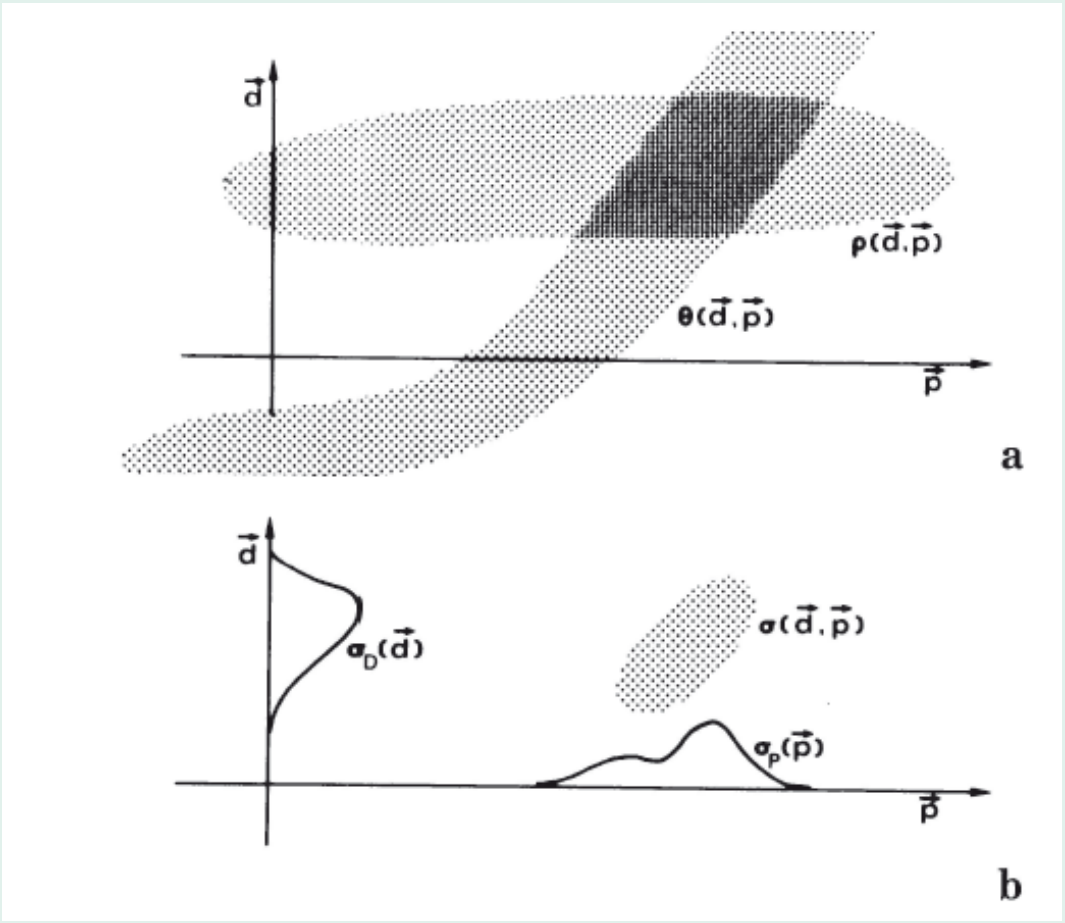






$N_{pix} = \mathcal{N}_{tot}$; $n_{*/pix} = 1$ (~CMD)

d: data
p: model
e: uncertainty



I.- $\epsilon_{\text{data}} = \epsilon_{\text{model}} = 0$

$$\vec{p} = G^{-1}(\vec{d})$$

II.- Optimización: $\epsilon_{\text{data}} \gg \epsilon_{\text{model}}$

$$\chi^2 = (\mathbf{x} - \langle \mathbf{x} \rangle)^{\mathrm{T}} \, \mathbf{C}^{-1} \, (\mathbf{x} - \langle \mathbf{x} \rangle)$$

III.- Inverse problem $\epsilon_{\text{data}} \sim \epsilon_{\text{model}}$
$\epsilon_{\text{data}} < \epsilon_{\text{model}}$

Look for additional information,
Re-define analysis tools

Take extreme care in inferences!!

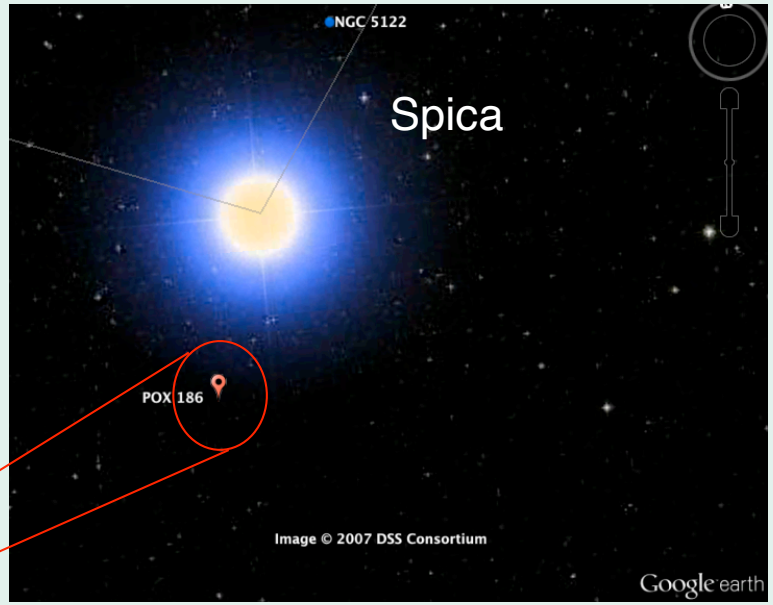
Any case:

Characterize model "uncertainty"

Tarantola, A., & Valette, B. (1982). Inverse Problems = Quest for Information. Journal of Geophysics, 50, 159.

~1995: IUE observation of Pox 186



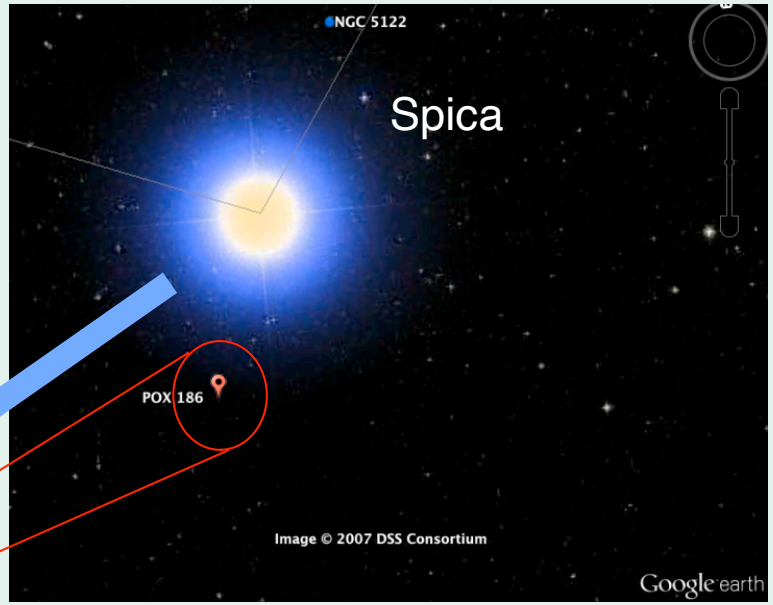


We observe Pox 186, we fit it with synthesis models (a very good fit), although the age was a bit older than expected



~1995: IUE observation of Pox 186





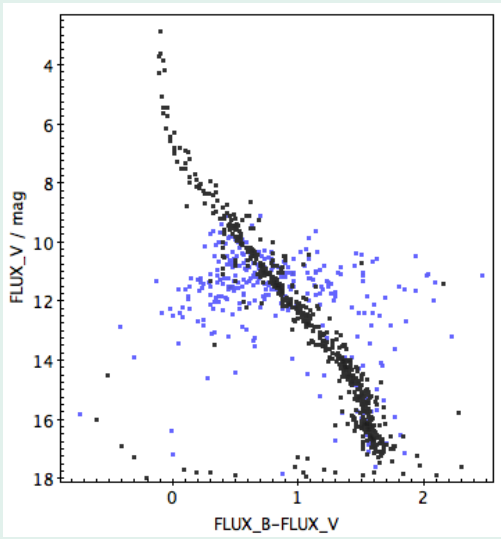
We observe Pox 186, we fit it with synthesis models (a very good fit), although the age was a bit older than expected

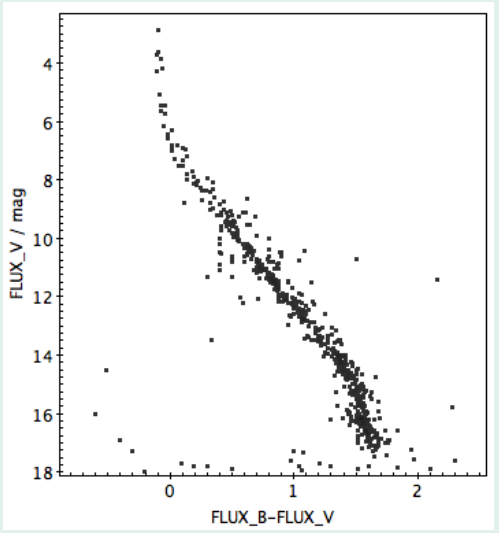


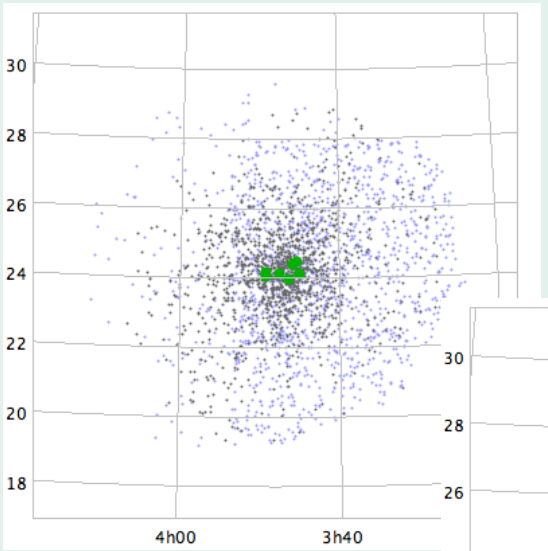
But the actual observation, was not Pox 186; BUT, mainly, diffuse light from Spica!!! (the age was compatible with a cluster dominated by Spica-like stars)

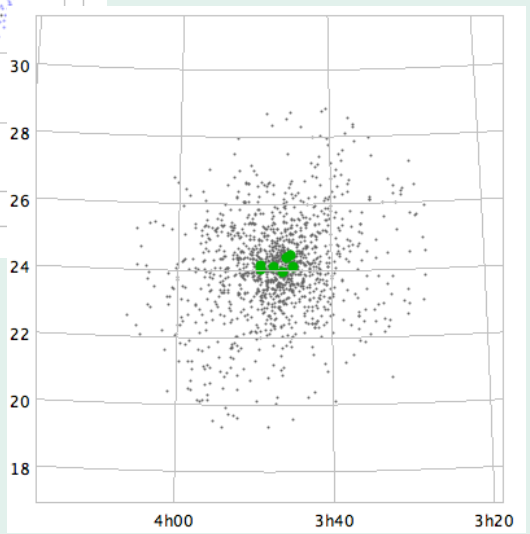
$< \ell_\lambda(t) >_{\mathcal{N}}$ vs. $< \ell_\lambda(t) >$



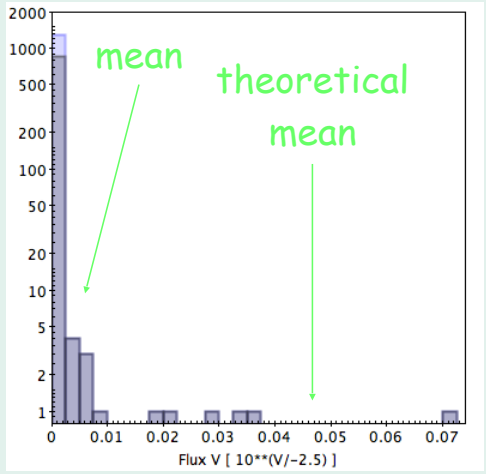


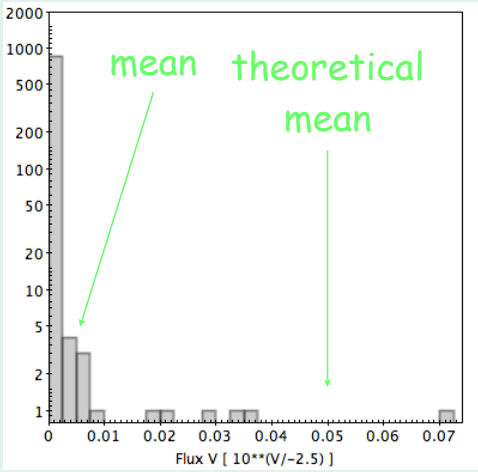






Pleiades





real luminosity star-by-star distributions:
In “most” of small clusters there is a deficit of luminous post main sequence stars… (i.e. kurtosis: outliers are the ones with luminous stars!)

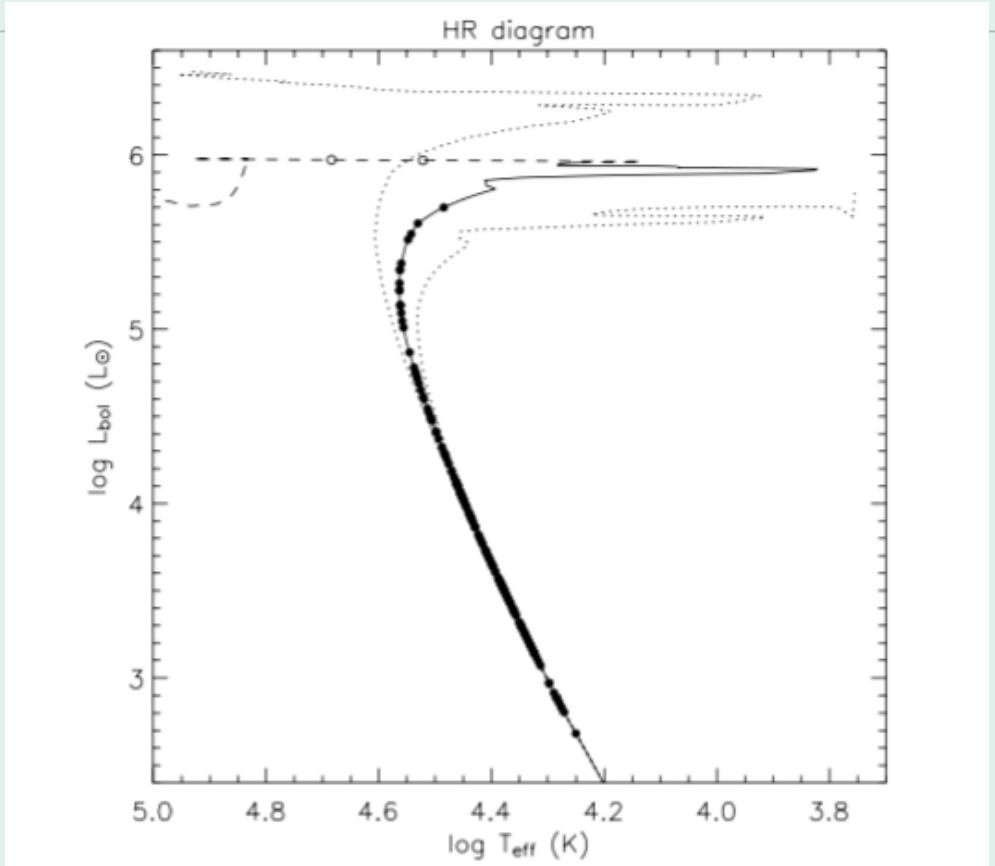


Fig. 15. Final adopted HR diagram for all the detected stars. The circles show the selected individual stellar model. The dashed part of the diagram is the WR branch. The two dotted curves represent the HR diagram at 3.0 Myr (upper curve) and 5.5 Myr (lower curve).

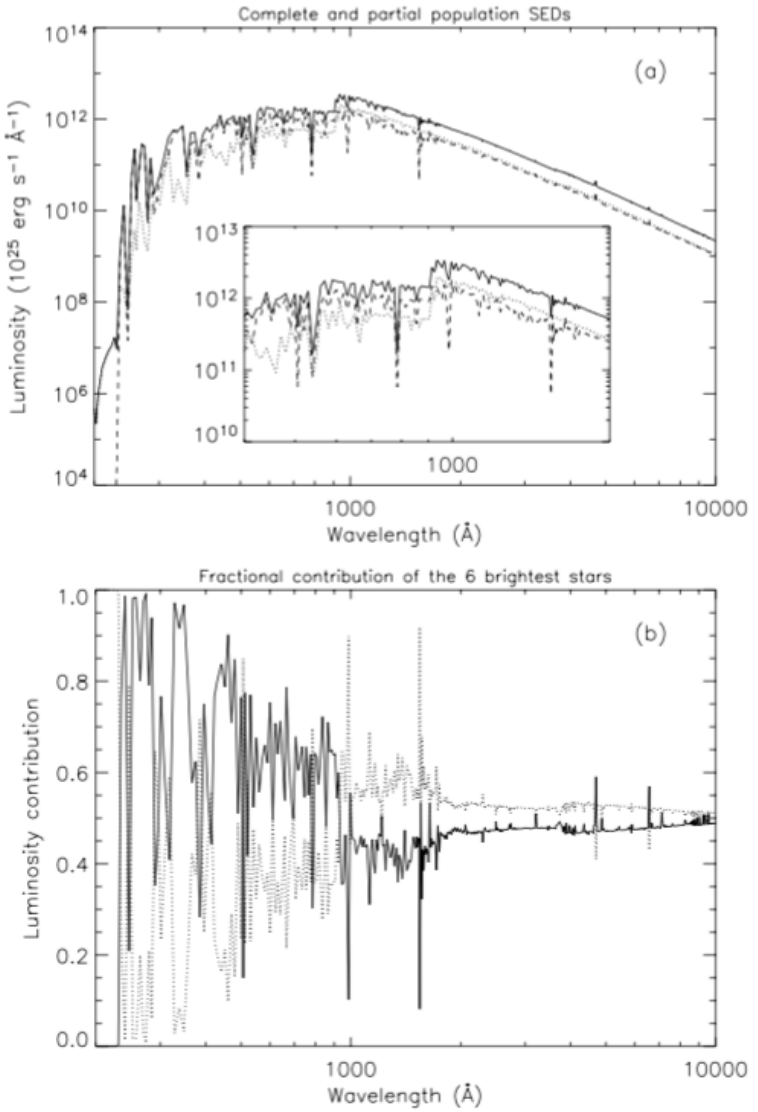


Fig. 19. *Top*: SEDs of the whole cluster (full line), of the 6 brightest stars (dashed line) and of the remaining 167 stars (dotted line). *Bottom*: ratio of the brightest 6 stars SED (full line) and of the remaining 167 star SED (dotted line) to the total SED.

Jamet et al 2004

What are SBFs? (the right way)

Tonry & Schneider 1988, AJ 93, 807

The idea:

larger distances ⇒ less "local"

details

(lower surface fluctuations)

but at the same time:

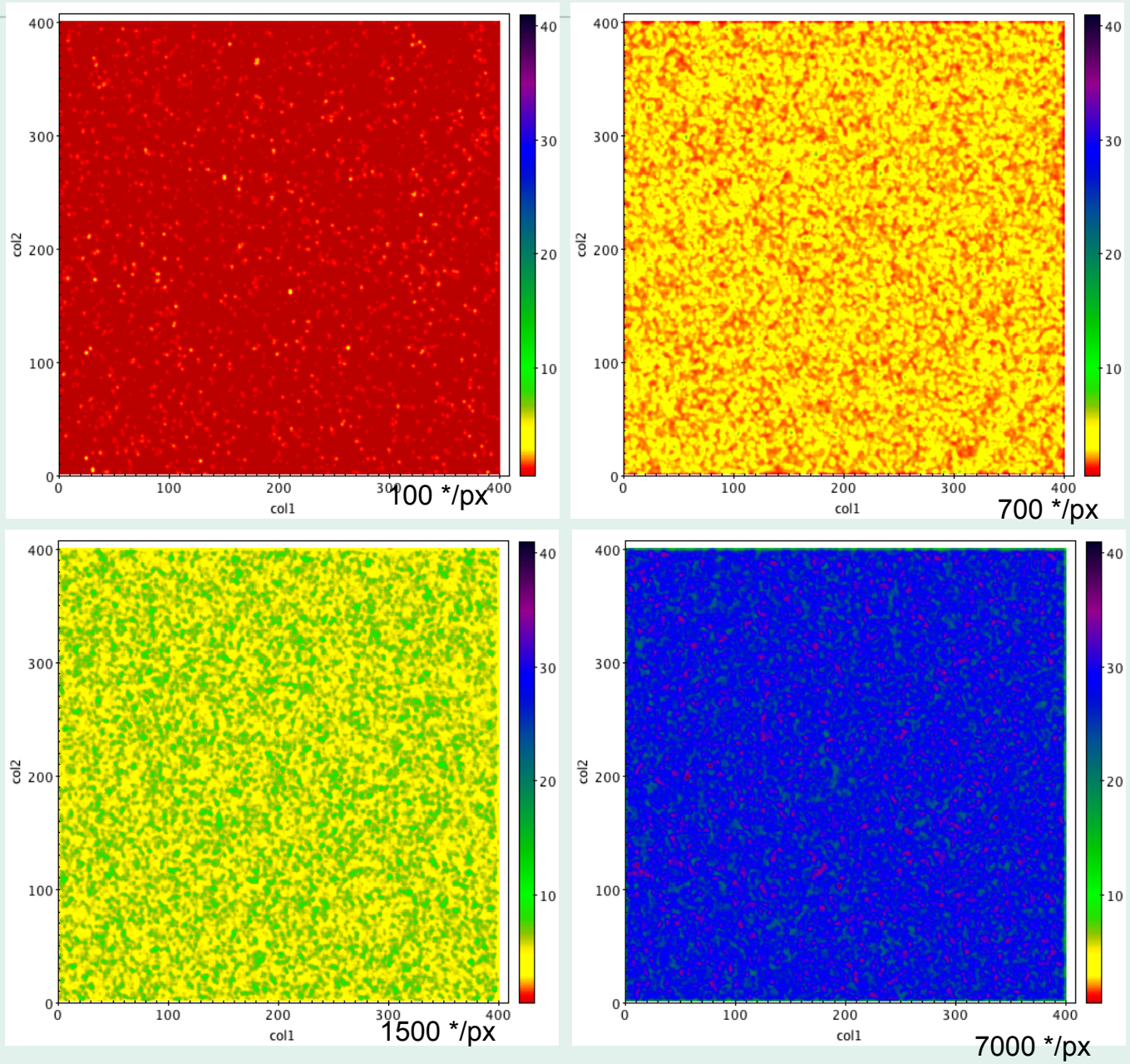
$$\frac{\sigma^2(\mathscr{L}, n_{*/\mathrm{pix}})}{\mu_1'(\mathscr{L}, n_{*/\mathrm{pix}})} = \mathrm{cte} = \frac{\sigma^2(\ell)}{\mu_1'(\ell)} = \bar{\ell}$$

Given that:

$$\sigma^2(\mathscr{L}_\lambda) = (4\pi\, d^2)^2\, \sigma^2(f_\lambda)$$

$$\mu_1'(\mathscr{L}_\lambda) = \quad 4\pi\, d^2 \quad \mu_1'(f_\lambda)$$

$$\bar{\mathscr{L}}_\lambda = \bar{\ell}_\lambda = 4\pi\, d^2\, \bar{f}_\lambda$$



Note: distribution of 1600 MC simulations in the plane for Age and Z assumed for NGC 1052-DF2

The method

Tonry & Schneider 1988, AJ 93, 807

1.- Obtain the fluctuation image

Produce an image of fluctuations as

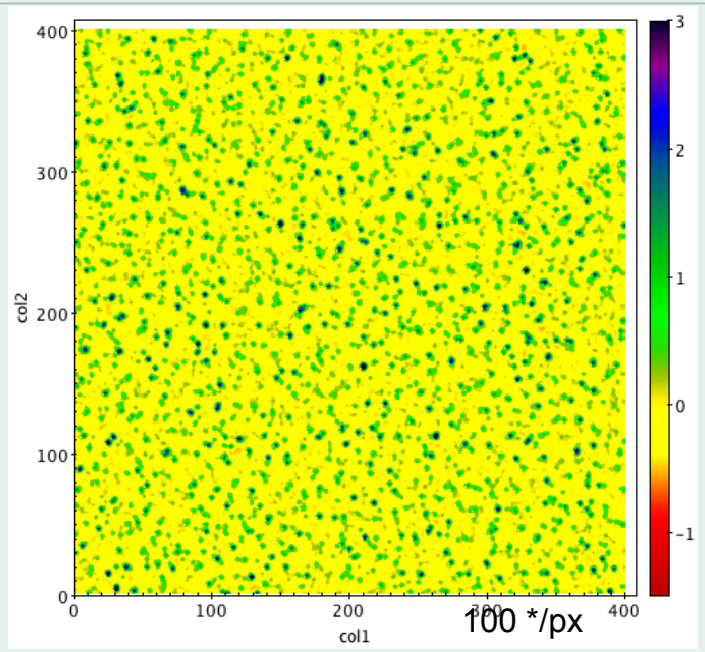
$$f(x,y) \rightarrow \frac{f(x,y) - < f(x,y) >_{\rm local}}{\sqrt{< f(x,y) >_{\rm local}}}$$

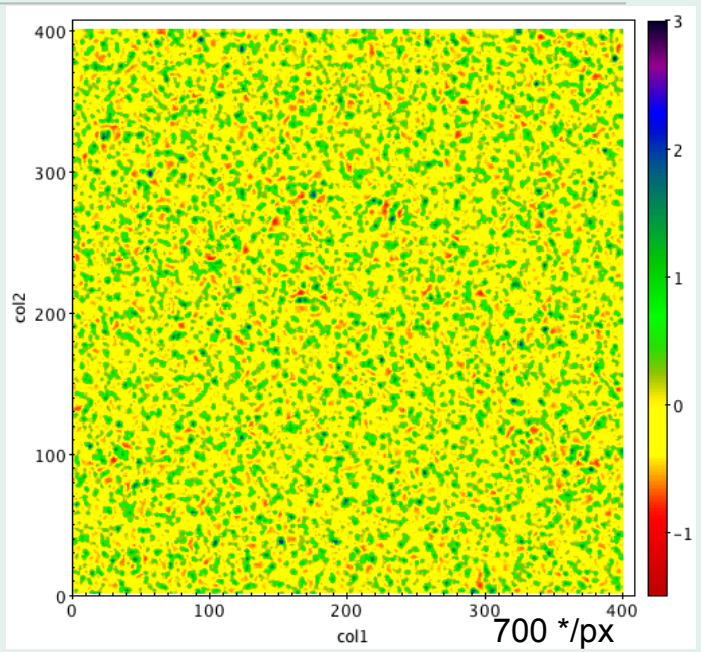
Note: "**local**" mean is needed to avoid large scale variations in $\boldsymbol{f}(x,y)$ due to the galaxy profile.

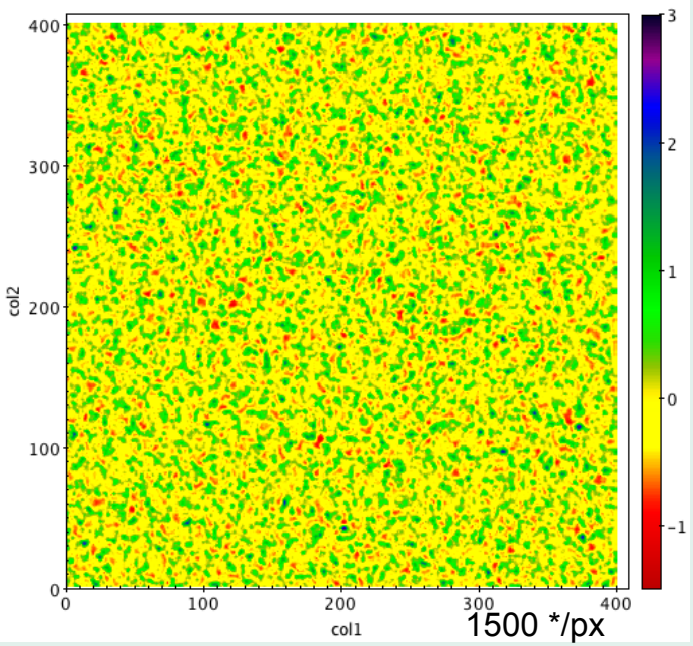
Usually it is needed to clean the image from GC, and possible foreground galaxies or stars

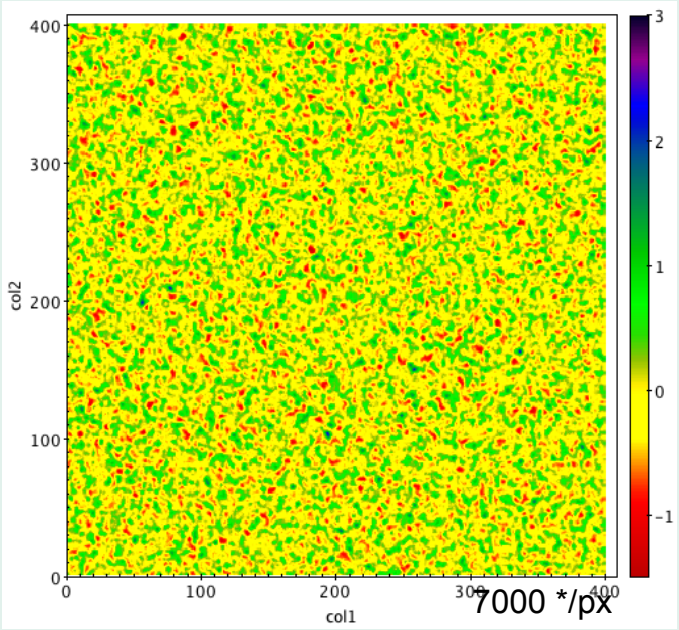
II.- Obtain the variance of the fluctuation image

Note: It is done using the power spectrum of the image to discriminate between the noise and the signal (the last is convolved with the PSF)









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The wokshop has been funded by grant PID2019-107408GB-C41

all grants are funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe"